PRACTICAL

GEOMETRY,

FOR THE USE OF THE

ROYAL MILITARY ACADEMY,

AT

WOOLWICH.

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PREFACE.

IN composing this treatise of practical geometry, it has not been my intention to explain or demonstrate the principles of the different branches of science upon which it is founded, this having been already done, by Authors who are well known, and whose works are in the hands of the Publick; but my chief views have been to collect together such parts as I thought most necessary for those who make the art of war their professional study, and to apply them practically, in as concise a manner as is consistent with perspicuity, in order that they may easily comprehend all that is essential to be known, and readily recollect, or apply what they may have been taught before. Those whose employment it is to teach the mathematical sciences, will readily agree, that nothing contributes so much to strengthen the mind, and assist the memory of a learner, as to have it in his power, to find, in an abridgment, the substance of what he has pre-

viously learnt, as this soon brings again to his recollection the knowledge he may have lost, and by degrees enables him to acquire a habit of studying, that, in the end, will lead him to make more use of his judgment than of his memory; which, in all cases, where it is required to direct the mind to any new subject, will be found a matter of the utmost importance.

As I intend to publish hereafter, a treatise of practical geometry on the ground, in which the application of trigonometry will be given, I have omitted that part of the science in this treatise; and have only to hope, that the various problems which I have here collected, with great care, and assiduity, will be found to answer the purpose for which they were intended.

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PRACTICAL GEOMETRY.

DEFINITIONS.

1. *A Point*, is that which has no parts, or magnitude.

2. *A line*, is that which has length, without breadth or thickness, as A B, fig. 1. Pl. 1. Lines are of three kinds, straight lines, curved lines and mixed lines.

3. *A straight or right line*, is the shortest that can be drawn from one point to another, as A B, fig. 1. Pl. 1.

4. *A curve*, is a line which continually changes its direction from one point to another, as C D, fig. 2. Pl. 1.

5. *A mixed line*, is composed of a straight line and a curve, as B C D, fig. 3. Pl. 1.

6. *A surface or superficies*, is that which has length and breadth without thickness, as M, fig. 4. Pl. 1.

7. *A solid*, is that which has length, breadth and thickness, as B, fig. 5. Pl. 1.

B 8. *A per-*

8. *A perpendicular*, is a line C D, which stands on another line A B, and does not incline more to one side of it than to the other, as fig. 6. Pl. I.

9. *A tangent*, is a line which touches a circle, or any other curve, without cutting it, as H T; and the point C, where the line H T touches the arc A C B, is called the *point of contact*, fig. 7. Pl. I.

10. *A secant*, is a line which cuts a circle, or any other curve, as A B, fig. 8. Pl. I.

11. *Parallel lines*, are those which have no inclination to each other, being every where equidistant, though ever so far produced, as A B, C D, fig. 9, and E F, G H, fig. 10. Pl. I.

12. *An angle*, is the inclination of two lines, A B, B C, which meet in a point B, called *the vertex*, or *angular point*; and the two lines, A B, B C, are called *the legs*, or *sides* of the angle B, fig. 11. Pl. I.

13. When several lines proceed from the same point, forming different angles, it is necessary to make use of three letters to distinguish them from each other, always placing that letter in the middle which denotes the vertex, as A B C, C B D, or A B D, fig. 12. Pl. I.

14. *A rectilinear*, or *right lined angle*, is that whose

whose legs or sides are right lines, as A B C, fig. 11. Pl. 1.

15. *A curvilinear angle*, is that whose legs or sides are curves, as B, fig. 13. Pl. 1.

16. *A mixtilinear angle*, is that which is composed of a right line and a curve, as C, fig. 14. Pl. 1.

17. The measure of a rectilinear angle F B H is the arc F H contained between its sides B F, B H, fig. 15. Pl. 1.

18. *A right angle*, is that which is formed by one line being perpendicular to another, as C A B, the measure of which is an arc C B of 90 degrees, fig. 16, Pl. 1.

19. *An acute angle*, is that which is less than a right angle, or whose measure is less than 90 degrees, as D E F, fig. 17. Pl. 1.

20. *An obtuse angle*, is that which is greater than a right angle, or whose measure exceeds 90 degrees, as B E F, fig. 17. Pl. 1.

26. An angle which is either acute or obtuse, is called an *oblique angle*.

22. The *complement of an arc* or *angle*, is what it wants of 90 degrees; thus the arc A C, or the angle A B C, is the complement of the arc A D, or of the angle A B D, fig. 18. Pl. 1.

23. *The supplement of an arc*, is what it wants of a semicircle, or of 180 degrees; thus the

8. *A perpendicular*, is a line C D, which stands on another line A B, and does not incline more to one side of it than to the other, as fig. 6. Pl. I.

9. *A tangent*, is a line which touches a circle, or any other curve, without cutting it, as H T; and the point C, where the line H T touches the arc A C B, is called the *point of contact*, fig. 7. Pl. I.

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whose legs or sides are right lines, as A B C, fig. 11. Pl. 1.

15. *A curvilinear angle*, is that whose legs or sides are curves, as B, fig. 13. Pl. 1.

16. *A mixtilinear angle*, is that which is composed of a right line and a curve, as C, fig. 14. Pl. 1.

17. The measure of a rectilinear angle F B H is the arc F H contained between its sides B F, B H, fig. 15. Pl. 1.

18. *A right angle*, is that which is formed by one line being perpendicular to another, as C A B, the measure of which is an arc C B of 90 degrees, fig. 16, Pl. 1.

19. *An acute angle*, is that which is less than a right angle, or whose measure is less than 90 degrees, as D E F, fig. 17. Pl. 1.

20. *An obtuse angle*, is that which is greater than a right angle, or whose measure exceeds 90 degrees, as B E F, fig. 17. Pl. 1.

26. An angle which is either acute or obtuse, is called an *oblique angle*.

22. The *complement of an arc* or *angle*, is what it wants of 90 degrees; thus the arc A C, or the angle A B C, is the complement of the arc A D, or of the angle A B D, fig. 18. Pl. 1.

23. *The supplement of an arc*, is what it wants of a semicircle, or of 180 degrees; thus the

arc EF is the supplement of the arc FD, fig. 19. Pl. I.

24. A *circle*, is a plane figure bounded by a curve line ABCDA, called the *circumference*, and which is every where equally distant from a given point O, called the *centre*, fig. 20. Pl. I.

25. The *radius of a circle*, is a right line, ON, drawn from the centre to the circumference, fig. 21. Pl. I.

26. The *diameter of a circle*, is a right line AB passing through the centre O and terminated by the circumference, fig. 22. Pl. I.

27. An *arc* is any part of the circumference of a circle, as AB, fig. 23. Pl. I.

28. A *chord* or *subtense*, is a right line AB, joining the extremities of an arc AEB, fig. 24. Pl. I.

29. A *semicircle*, is that part of a circle which is contained between the diameter AB and half the circumference ACB, fig. 25. Pl. I.

30. A *quadrant*, is the fourth part of a circle, or that which is contained between two radii and an arc of 90 degrees, as H, fig. 26. Pl. I.

31 The terms *circle*, *semicircle* and *quadrant* sometimes denote the entire figures, and sometimes only the arcs by which they are bounded

32. A *segment*

32. A *segment of a circle*, is that part of a circle which is cut off by a chord, as A B E, fig. 24. Pl. I.

33. A *sector of a circle*, is that part of a circle which is contained between two radii C A, C B, and the arc A B, fig. 27. Pl. I.

34. The circumference of a circle is supposed to be divided into 360 equal parts, called *degrees*; each degree into 60 equal parts, called *minutes*; and each minute into 60 equal parts called *seconds*, &c. and these divisions are thus distinguished 32°. 26′. 15″, that is, 32 degrees, 26 minutes, and 15 seconds.

35. Plane figures bounded by three right lines, are called *triangles*.

36. A triangle which has its three sides equal, is called an *equilateral triangle*, as A B C, fig. 28, Pl. I.

37. A triangle which has only two equal sides, is called an *isosceles triangle*, as D E F, fig. 29. Pl. I.

38. A triangle which has all its sides unequal, is called a *scalene triangle*.

39. A triangle which has a right angle, is called a *rectangular*, or *right-angled triangle*, as B A C; and the side B C opposite to the right angle A, is called the hypothenuse, fig. 30. Pl. I.

40. A triangle which has an obtuse angle, is called an *obtuse angled*, or *ambligonal triangle* as DEF, fig. 31. Pl. 1.

41. A triangle which has all its angles acute, is called an *acute-angled*, or *oxigonal triangle*, as ABC, fig. 28. Pl. 1.

42. A triangle which has no right angle, is called an *oblique triangle*, as fig. 28, 29, 31. Pl. 1.

43. The three angles of any plane triangle, taken together, are equal to two right angles, or 180 degrees.

44. The *height*, or *altitude* of a figure, is a perpendicular AD let fall from any one of the angles to its opposite side BC, called the *base*, fig. 32. Pl. 1. or to the prolongation of the base, as in fig. 33.

45. Plane figures, bounded by four right lines, are called *quadrangles*, or *quadrilaterals*.

46. A *square*, is a quadrilateral, having all its sides equal and all its angles right angles, as A, fig. 34. Pl. 1.

47. A *parallelogram*, is a quadrilateral, whose opposite sides are parallel, fig. 35, 36, 37. Pl. 1.

48. A *rectangle*, is a parallelogram, whose angles are all right angles, fig. 34 and 35. Pl. 1.

49. A *rhomboid*, is an oblique angled parallelogram, fig. 36. Pl. 1.

50. A *rhombus*, or *lozenge*, is a quadrilateral whose sides are all equal, but its angles oblique, as fig. 37. Pl. 1.

51. A *trapezium*, is a quadrilateral, which has none of its sides parallel to each other, fig. 38, Pl. 1.

52. A *trapezoid*, is a quadrilateral which has only two of its opposite sides parallel, fig. 39. Pl. 1.

53. A *diagonal*, is a right line joining any two opposite angles of a quadrangle, as A B, fig. 40. Pl. 2.

54. Plane figures, bounded by more than four sides, are called *polygons*.

55. A *regular polygon*, is that which has all its sides and angles equal, as fig. 42, 43, &c. Pl. 2.

56. An *irregular polygon*, is that which has its sides and angles unequal, as fig. 41. Pl. 2.

57. Polygons have particular names according to the number of their sides; thus, a polygon of 5 sides, whether regular or irregular, is called a *pentagon*; of 6 sides, an *hexagon*; of 7 sides, an *heptagon*; of 8 an *octagon*; of 9 a nonagon; of 10 a decagon; of 11 an undecagon; and of 12 a dodecagon, as fig. 42, 43, 44, 45, 46, 47, 48, and 49. Pl. 2.

58. *Similar figures*, are such as have all their angles A, B, C, D, E, a, b, c, d, e, respec-

tively equal, each to each, and their sides A B, B C, C D, &c. a b, b c, c d, &c. about the equal angles proportional, as fig. 50 and 51. Pl. 2.

59. *Homologous sides* are those sides in similar figures, which are proportional, or contiguous to equal angles; thus A B is homologous to a b, B C to b c, and so on. Fig. 50 and 51. Pl. 2.

60. *Corresponding angles*, are those angles in similar figures, which are contiguous to their homologous sides; as the angle A to the angle a; the angle B to the angle b, and so on. Fig. 50 and 51. Pl. 2.

PRACTICAL GEOMETRY.

SECT. I.

Lines, Angles, and Figures.

Prob. 1.

To divide a given right line A B *into two equal parts*. fig. 1. Pl. 3.

Method 1. From A and B as centres, and with any radius greater than half A B, describe arcs cutting each other at C and D. Through C and D draw a line C D, and E will be the point of bisection of the line A B.

Method 2. When the line is near the extreme edge of a plane. fig. 22. Pl. 3.

From A and B as centres, and with any radius, describe arcs intersecting each other in E; from the same centres A and B, with any radius less than the former, describe arcs cutting each other in D. Through E and D draw E C, which will divide A B into two equal parts.

Method

Method 3. *By the line of lines on the sector,* fig. 2. Pl. 3.

Take the length A B in the compasses. Open the sector till this extent forms a transverse distance between 10 and 10. Take the length from 5 to 5 on the same line, and it will be the half of A B as required.

Note. By this method, any line may be divided into 2, 4, 8, 16, 32, 64, 128, &c. equal parts, by successively dividing each subdivision into two.

Prob. 2.

To divide a given arc A B *into two equal parts,* fig. 3. Pl. 3.

From A and B as centres, with any opening in the compasses, more than half A B, describe arcs cutting each other at C and D. Through their intersections draw a line C D, and E will be the point of bisection.

Prob. 3.

To erect a perpendicular on a given line B C, *from a given point* A *in it,* fig. 4. Pl. 3.

Set off on each side of the point A any two equal distances A D, A E. From D and E as centres, and with any radius greater than half

D E

D E describe two arcs intersecting each other in F. Through the points A and F draw the line A F, and it will be the perpendicular required.

Prob. 4.

To erect a perpendicular at, or near the end of a given line C D, fig. 5. Pl. 3.

Method 1. From any point A as a centre, and with A D as a radius, describe an indefinite arc B D E. Through B and A draw the line B E, and join D E, which will be the perpendicular required.

Method 2. fig. 6. Pl. 3.

From the point B with any radius, describe an indefinite arc A C D. Set off the same radius A B on the arc A D, from A to C, and from C to D. From the points C and D, with any radius, describe arcs cutting each other in E. Through B and E draw the line B E, and it will be the perpendicular required.

Method 3. fig. 7. Pl. 3.

From C to D set off any length 4 times; from C as a centre, with 3 of the same parts, describe an arc at E, and from D, with 5 of them, cut the arc E. Through E and C draw the line C E, which will be the perpendicular required. This method is generally called raising a perpendicular by the numbers 3, 4, and 5.

Note.

Note. Any equimultiples of these numbers may be used for erecting a perpendicular at the end of a given line, as 6, 8, and 10; 9, 12, and 15, &c. which several lengths may be taken from any plane scale.

Method 4. fig. 8. Pl. 3.

From any point E, taken in the line A B, as a centre, and with E B as a radius, describe the indefinite arc B F. From B, with the same radius, cut B F in C; and from C as a centre, and with C B as a radius, describe the arc B D, on which set off the extent B C twice, that is, from B to G, and from G to D. Then join the points B and D, and it will give the perpendicular required.

Prob. 5.

From a point D, *out of a given line* A B, *to let fall a perpendicular,* fig. 9. Pl. 3.

Method 1. From D as a centre, and with any radius, describe an arc intersecting the given line. From the points of intersection C and E, with any radius, describe two arcs cutting each other at F. Then through D and F draw a line, and D G will be the perpendicular required.

Method 2. *When the point* E *is nearly opposite to the end of the line* A B, fig. 10. Pl. 3.

Through E draw a line cutting A B at any point C. Bisect C E; and from the point of bisection

bisection D as a centre, and with D E as radius, describe an arc E F C. Then join E F, and it will be the perpendicular required.

Method 3. fig. 11. Pl. 3.

From A, or any other point, in A B, with any radius A D, describe the arc D E, and from any point C, with the radius C D, describe another arc, cutting the former one in D and E. Then join D and E, by a line D F E, and D F will be the perpendicular required.

Prob. 6.

Through a given point C, *to draw a line parallel to a given line* A B, fig. 12. Pl. 3.

Method 1. From any point D in the line A B, as a centre, with the radius D C, describe the arc C E; and from C, with the same radius, describe the arc D F. Take E C in the compasses, and set it off from D to F. Though C and F, draw G H, which will be the parallel required.

Method 2. fig. 13. Pl. 3.

From the given point D, take the length D F, by describing from D as a centre, an arc to touch A B, without cutting it; and with the same length, from any point G in the line A B describe an arc H I. Then through D draw C E a tangent to the arc H I, and it will be the parallel required.

Method 3.

Method 3. *When the parallel line is to be at a given distance* M N, fig. 14. Pl. 3.

From any two points G and E in the line A B, as centres, and with M N as a radius, describe the arcs H and F. Draw C D touching these arcs, in H and F, and it will be the parallel required.

Method 4. *When the parallel is required to be drawn at a considerable distance from a given line*, fig. 15. Pl. 3.

From any two points G and H in the given line A B, erect the perpendiculars G C, H D, (Prob. 3. and 4.) on which set off from G to C, and from H to D the given distance. Then join C D, and it will be the parallel required.

Perpendiculars and parallel lines may be drawn by an instrument, called the German parallel ruler. It consists of a right angled triangle A B C, fig. 1. Pl. 4, commonly called a *square* or *angle*, and of a ruler D E; both made of wood, or ivory.

USE of the ANGLE and RULER.

Prob. 7.

To erect a perpendicular at any point C *in the given line* A B. fig. 2. Pl. 4.

Method 1. Apply one of the perpendicular sides E F, of the angle, upon the line A B. Lay the

the ruler H I againſt the side E G; and keeping it steady, slide the angle upwards, till the side F G touches the point C. Then draw C D, and it will be the perpendicular required.

Method 2. fig. 3. Pl. 4. Apply the longest side G E of the angle upon the line A B. Lay the ruler H I against the side G F. Keep it steady, and turn the angle so that the side F E may be laid against the ruler; then slide the angle upwards, till the long side of it touches the point C; after which draw C D, and it will be the perpendicular required.

Prob. 8.

Through a given point C *to draw a line parallel to a given line* A B, fig. 4 and 5. Pl. 4.

Place one of the edges of the angle D upon the line A B. Lay the ruler against one of the other edges of the angle, and keeping it steady, slide the angle till the same edge which had been placed upon the line A B, touches the point C; then through C draw E F, which will be the parallel required.

Note. With this instrument it is also easy to describe squares and parallelograms.

Prob. 9.

To divide a given line A B *into any number of equal parts*, fig. 6. Pl. 4. *for instance into* 5.

Method 1.

Method 1. Through A draw an indefinite line A M, making any angle with A B. Set off on this line, from A to G as many equal parts of any length as A B is to be divided into. Join B G, and parallel to it draw F L, E K, D I, C H, which will divide the line A B as was required.

Method 2. fig. 7. Pl. 4. From the end A of the given line A B, draw A C, making any angle with A B. Through the end B, draw B D parallel to A C. Set off from A to E as many equal parts less one, as A B is to be divided into. Set off the same number of these parts from B to F. Then draw the lines G E, H M, I L, F K, and they will divide A B as was required.

Method 3. fig. 8. Pl. 4. Through B draw C E, making any angle with A B. Take any point E, through which draw E D parallel to A B. From E to D, set off as many equal parts of any length, as A B is to be divided into. Through D and A draw D A, and produce it till it meets C E in C. Then lines drawn from the points F, G, H, I to the point C will divide the line A B into the required number of equal parts.

Method 4. fig. 9. Pl. 4. By the Sector.

Suppose it be required to divide the given line A B into seven equal parts. Multiply 7 by any number, for instance by 10, which will make 70. Make

Make A B a transverse distance between 70 and 70, on the line of lines; then keeping the sector thus opened, take with the compasses the transverse distance between 10 and 10, which will be the 7th part required.

Note. If the given line should be too long to be applied to the legs of the sector, divide one half, or one fourth of it, by 7, and the double, or quadruple of this 7th part, will divide the given line as required.

Prob. 10.

To cut off from a given line A B, *which is supposed to be very short, any proportional part*, fig. 10. Pl. 4.

Suppose for instance, $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, &c. should be required. From the ends A and B, draw A D, B C, perpendicular to A B. From A to D set off any opening of the compasses 12 times, and the same from B to C. Through the divisions 1, 2, 3, &c. draw lines 1 f, 2 g, &c. parallel to A B. Draw the diagonal A C, and 1 d will be the $\frac{1}{12}$ of A B; 2 C, $\frac{2}{12}$, and so on. The same method is made use of for obtaining any other proportional part of a given line.

Prob. 11.

To make a diagonal scale of feet, inches and tenths of an inch, fig. 11. Pl. 4.

Draw an indefinite line A B, on which set off from A to B the given length for one foot, any required number of times. From the divisions A, C, H, B, draw A D, C E, &c. perpendicular to A B. On A D and B F, set off any length ten times; through these divisions draw lines parallel to A B. Divide A C and D E into 12 equal parts; each of which will be one inch. Draw the lines A 1, G 2, &c. and they will form the scale required, (prob. 10.).

Note. A scale which has one of its subdivisions divided into 10 equal parts by a diagonal line, is called a *decimal scale*, and a *duodecimal scale* is one which is divided into 12 equal parts.

Prob. 12.

To find a third proportional to two given lines M *and* N, fig. 12. Pl. 4.

Draw two right lines making any angle D A E; in these lines take A C, equal to the first term M; and A E, A B, each equal to the second term N: join B, C, and through E draw E D parallel to C B; then A D will be the third proportional required.

That is, A C : A E :: A B : A D.

or, M : N :: N : A D.

By the Sector.

Case 1. *When the proportion is increasing*, fig. 13. Pl. 4.

Open

Open the sector, and make A B a transverse distance between 20 and 20, on the line of lines marked L. With the same opening of the sector, find the transverse measure of C D, which suppose 30. Make C D a transverse distance between 20 and 20; then take the extent between 30 and 30, and it will be the third proportional E F required.

That is, A B : C D : : C D : E F.

Case 2. *When the proportion is decreasing*, fig. 14. Pl. 4.

Open the sector, and make A B a transverse distance between 100 and 100; with the same opening of the sector, find the transverse distance C D, which suppose 70. Make C D a transverse distance between 100 and 100. Then take the extent between 70 and 70, and it will be the third proportional E F required.

That is, A B : C D : : C D : E F.

Note. When the lines of the first and second term are too small, they may be doubled, or tripled; and having found the third term, the half, or third of it, must be used accordingly. And when the lines of the first and second term are too extensive, one half or one third of them may be taken; and having found the third proportional, it is to be doubled or tripled accordingly.

Prob. 13.

To find a fourth proportional to three given lines, M, N, O, fig. 15. Pl. 4.

Draw two lines, making any angle whatever, as DAE. In these lines take AB equal to the first term M; AC equal to the second N; and AD equal to the third O. Join BC, and through the point D draw DE parallel to BC, and AE will be the fourth proportional required.

That is, AB : AC :: AD : AE.
or, M : N :: O : AE.

By the Sector.

Case 1. *When the proportion is increasing,* fig. 16. Pl. 4.

Make the first term AB a transverse distance between 20 and 20, on the line of lines. Find the transverse measure of the second term CD, which suppose to be 30. Make the third term EF, a transverse distance between 20 and 20. Then the measure between 30 and 30, will be the fourth proportional MN required.

That is, AB : CD :: EF : MN.

Case 2. *When the proportion is decreasing,* fig. 17. Pl. 4.

Make the first term AB a transverse distance between 100 and 100; find the transverse mea-

sure

sure of the second term C D, which suppose is 70. Make the third term E F a transverse distance between 100 and 100. Then the measure between 70 and 70, will be the fourth proportional M N required.

That is, A B : C D :: E F : M N.

Prob. 14.

To find a mean proportional between two lines, A *and* B, fig. 1. Pl. 5.

Draw any line, in which take C D, equal to A, and D E equal to B. Bisect C E in F, and with F C as radius, describe the semicircle C G E. From the point D draw D G perpendicular to C E, and D G will be the mean proportional required.

That is, C D : D G :: D G : D E.
or, A : D G :: D G : B.

Prob. 15.

To find a mean proportional between the extremes, A B, B C, fig. 2. Pl. 5.

Bisect A B, and from the point of bisection F, as a centre, and with F A, or F B as a radius, describe the semicircle A E B. From C draw C E perpendicular to A B, and join E B, which will be the mean proportional required.

That is, B C : B E :: B E : A B.

Prob. 16.

To divide a line A B *into extreme and mean proportion*, fig. 3. Pl. 5.

Method 1.

Draw the line A D perpendicular to A B, and make it equal to half A B. Join D B, and from D as a centre, with D A as a radius, describe an arc cutting B D in E. From B as a centre, and B E as radius, describe an arc intersecting A B in C, which will divide the line A B according to the required proportion.

That is, A B : B C :: B C : A C.

Method 2. *By the sector*, fig. 4. Pl. 5.

Case 1. Make A B a transverse distance between 60 and 60, on the line of chords. Then take the transverse distance between 36 and 36, and set it off from B to C, which will divide the line A B as was required.

That is, A B : B C :: B C : A C.

Prob. 17.

To divide a line A B *in the same proportion as another given line*, C D. fig. 5, Pl. 5.

Draw A H, making any angle with A B. Upon A H set off the several divisions of C D. Join H B, and parallel to it draw the lines L M, K N, I O; and A B will be divided as was required.

Prob.

Prob. 18.

To draw a tangent to a given circle, passing through a given point A.

Case 1. *When the point* A *is in the circumference of the circle,* fig. 6. Pl. 5.

From the center O draw the radius O A. Then through the point A, draw C D perpendicular to O A, and it will be the tangent required.

Case 2. *When the point* A *is without the circle,* fig. 7. Pl. 5.

From the centre O draw O A, and bisect it in F. From the point F, with F A, or F O, as a radius, describe the semicircle A D O, cutting the given circle in D. Then through the points A and D, draw A B, and it will be the tangent required.

Prob. 19.

To find the point where a line A B *touches the circumference of a given circle,* fig. 8. Pl. 5.

From the centre C let fall the perpendicular C E upon A B, and the intersection D will be the point of contact required.

Prob. 20.

To divide a given angle A B C *into two equal parts,* fig. 9. Pl. 5.

From B as a centre, with any radius, describe an arc A C. From A and C, with any radius, describe arcs intersecting each other in D. Then draw B D, and it will bisect the angle, as required.

Prob. 21.

To divide a right angle A B C *into three equal parts.* fig. 10. Pl. 5.

From B as a centre, with any radius, describe the arc A C. From A, with the radius A B, cut the arc A C in D; and with the same radius from C cut it in E. Then through the intersections D and E, draw the lines B D, B E, and they will trisect the angle, as was required.

Prob. 22.

To divide any given angle, A B C, *into three equal parts,* fig. 11. Pl. 5.

From B, with any radius, describe the circle A C D A. Bisect the angle A B C, and produce A B to D. On the edge of a ruler mark off the length of the radius A B. Lay the ruler on D, and move it till one of the marks on the edge intersects B E, and the other the arc A C in G. Set off the distance C G from G to F; and draw the lines B F, B G, and they will trisect the angle A B C.

The construction of an instrument to divide a given angle into any number of equal parts, fig. 12. Pl. 5.

This

This instrument is composed of a curve A S and a right angle A B S, made of a thin plate of brass, horn, or paste-board, and the curve, which is called the *quadratrix of Tschirnhausen*, is described as follows.

Draw B A perpendicular to B T, and with any radius describe the quadrant A T. Divide the quadrant into any number of equal parts, and the radius A B into the same number of equal parts. From the divisions G, H, I, K, &c. draw lines parallel to B T; and from the points C, D, E, F, &c. draw the lines C B, D B, E B, &c. Then through the intersections L, M, N, O, &c. describe the curve A S, and it will be the one required.

Prob. 23.

To divide a given acute angle I K L, *into five equal parts; by the quadratrix*, fig. 13. Pl. 5.

Case 1. Apply the side A B of the quadratrix upon I K, the point B corresponding with the angle K. Draw a line along the curve A S, cutting K L in F. Remove the instrument, and from F let fall the perpendicular F E upon I K. Divide E I into five equal parts, and through the points of division, draw C M, D N, &c. parallel to E F. Then from their intersections, M, N, O, P, draw the lines K M, K N, K O, K P, and they will divide

divide the angle I K L, into the number of equal parts required.

Case 2. *When the given angle is an obtuse angle,* A B C, fig. 14. Pl. 5.

From B, with any radius, describe the arc A C. Bisect the angle A B C, and divide one of the halves as A B D, into five equal parts, by the quadratrix. Set off the distance A E (being $\frac{2}{5}$ of the angle A B D) from E to F, from F to G, and from G to H. Then through E, F, G, H, draw lines B E, B F, B G, B H, and they will divide the angle A B C into the number of equal parts required.

Prob. 24.

At the point D *to make an angle* E D F *equal to a given angle* A B C, fig. 15. Pl. 5.

From B, with any opening in the compasses, describe the arc C A. From D, with the same radius, describe the arc E F. Take the length C A, and set it off from E to F. Then through F, draw the line D F, and the angle E D F, will be equal to the angle A B C, as was required.

Prob. 25.

At the point C, *in a given line* A B, *to lay down an angle of any number of degrees,* fig. 16. Pl. 5.

Method

Method 1. *By the protractor.*

Suppose the given angle to be of 55 degrees. Apply the diameter of the protractor to the line A B, so that the centre may coincide exactly with the point C. Make a mark against 55 on the edge of the protractor at D. Then remove the instrument, and draw a line from C through the point D, and the angle B C D will contain the number of degrees required.

Method 2. *By the line of chords on the protracting scale,* fig. 17. Pl. 5.

Take the first 60 divisions, or degrees from the line of chords, and from the point C, with this distance, describe the arc B D. Take the length of 55 degrees from the same line of chords, and set it off from B to D. Then draw the line C D, and the angle B C D, will contain the number of degrees required.

Note. If the number of degrees are more than 90°, they are to be set off at twice.

Thus, should the angle A B C, fig. 1. Pl. 6. be of 120 degrees. From B, with 60 degrees, describe the arc A D C, on which set off 60 dedrees twice; that is from A to D and from D to C. Then through C draw A B, and the angle A B C will be the one required.

If the angle D E F. fig. 2. Pl. 6. should be of an odd number of degrees, as for instance 107.

After

After having described an arc D F with 60 degrees, set off from D to G 60 degrees, and from G to F, 47 degrees, (the difference between 60 and 107). Then through F draw the line E F, and D E F will be the angle required.

Method 3. *By the line of chords on the sector,* fig. 3. Pl. 6.

Case 1. When the given degrees are under 60°. as for instance 40°. From A with any radius A B, describe an arc B E. Open the sector, till the same radius A B makes a transverse distance between 60 and 60, on the line of chords. Take with the compasses the transverse distance from 40 to 40 on the same line of chords, and set it off from B to C. Then through C draw C A, and B A C will be the angle required.

Case 2. *When the given degrees are more than* 60°, fig. 4. Pl. 6.

Open the sector and describe an arc B E as before. Take $\frac{1}{2}$, or $\frac{1}{3}$ of the given number of degrees, and set them off from B to C and from C to D; that is twice if the degrees were halved, or three times if the third part was used as a transverse distance.

Case 3. *When the given angle is less than* 6° suppose 3°, fig. 5. Pl. 6.

From A with any radius A B, describe an arc B C, and set off the same radius from B to C.

Open

Open the sector as before, and take the transverse distance from 57 to 57, and set it off from C to D. Draw A D, and the arc D B, or the angle B A D, will contain the given number of degrees.

Prob. 26.

To find the number of degrees contained in a given angle B C D, fig. 16. Pl. 5.

Method 1. *By the protractor.*

Apply the diameter of the protractor to the line C B; so that its centre may coincide exactly with the angular point C; then the degree upon the edge, cut by the line C D, will be the measure of the given angle.

Method 2. *By the line of chords on the protracting scale*, fig. 17. Pl. 5.

Case 1. *When the angle is less than* 90 *degrees.*

From the angular point C, with the chord of 60 degrees, describe the arc B D. Then take the distance B D, and apply it to the line of chords; and it will shew the number of degrees contained in the angle.

Case 2. *When the angle is more than* 90 *degrees*, fig. 2. Pl. 6.

From the angular point E with the radius of 60 degrees, describe the arc D F. Set off the same radius from D to G, and measure the remainder G F on the line of chords, which being added

added to D G, or 60°, will give the number of degrees contained in the given angle.

Method 3. *By the line of chords on the sector,* fig. 3. Pl. 6.

Case 1. *When the given angle* B A C, *is less than* 60 *degrees.*

From the angular point A, with any radius, describe the arc B E. Open the sector, and make A B a transverse distance between 60 and 60. Take the length B C, and measure it on the same line of chords, which will shew the number of degrees contained in the given angle.

Case 2. *When the given angle* D E F *is more than* 60 *degrees,* fig. 2. Pl. 6.

From the angular point E, with any radius, describe the arc D F. Set off the radius E D from D to G. Open the sector, and make the same radius E D a transverse distance between 60 and 60. Take the remainder G F, and measure it on the same line of chords, which add to the arc D G, and their sum will be the number of degrees contained in the given angle.

Case 3. *When the given angle* B A D *appears to be less than* 6 *degrees,* fig. 5. Pl. 6.

From the angular point A, with any radius, describe an arc B C. Set off the radius A B from

B to

B to C. Open the sector, and make the same radius A B a transverse distance between 60 and 60. Measure the length D C on the same line of chords, which being deducted from 60 degrees, the remainder will be the number of degrees in the given angle.

Prob. 27.

On a given chord A B, *to describe an arc of a circle that shall contain any number of degrees, without compasses, or without finding the centre of the circle,* fig. 6. Pl. 6.

Method 1. Draw A C making any angle with A B. At any point C in A C, make the angle A C I, equal to the given angle. Through B draw B D parallel to C I, and the intersection D will be one of the points of the required arc. In the same manner as many other points F, H, &c. may be found, as will be necessary to complete the arc.

Method 2. *By which an arc may be described mechanically on a given chord* A B, fig. 7. Pl. 6.

Place two rulers, forming an angle A C B, equal to the supplement of half the given number of degrees; and fix them in C. Place two pins at the extremities of the given chord, and hold a pencil in C; then move the edges of this instrument

strument against the pins, and the pencil will describe the arc required.

Suppose it is required to describe an arc of 50 degrees on the given chord A B; subtract 25 degrees (which is half the given angle) from 180, and the difference, 155 degrees, will be the supplement. Then form an angle A C B of 155° with the two rulers, and proceed as has been shewn above.

Prob. 28.

On a given line A B *to describe a segment of a circle, capable of containing a given angle*, fig. 8. Pl. 6.

Bisect A B by the perpendicular E D. Make the angles E A B, F A B, G A B, H A B, respectively equal to the difference of the given angles and 90 degrees; observing that the angle must be made on the same side with the segment, if the given angle be less than 90 degrees; but on the opposite side, if the angle is greater than 90 degrees. Then the intersections E, F, G, H, will be the centres of the given segments.

Case I. *When the angle is less than* 90 *degrees, for instance* 70, fig. 9. Pl. 6.

Make the angle B A G equal to 20 degrees, and from the intersection G with G A describe the

the arc A C B. Then any angle as C, made in that segment, will be equal to 70 degrees.

Case 2. *When the angle is greater than* 90 *degrees, for instance* 120, fig. 10. Pl. 6.

Make the angle B A E equal to 30 degrees, and from the intersection E, with E A, as radius, describe the arc A F B. Then any angle as F made in that segment, will be equal to 120 degrees.

Another method, fig. 11. Pl. 6.

Make the angles B A D, A B D each equal to the given angle. To A D, B D, draw the perpendiculars, A O, B O. From their intersection O, with O A, or O B, as radius, describe the arc A C C B. Then any angle as C, made in that segment, will be equal to the given angle.

Prob. 29.

To find the centre of a given circle A C B D E A, fig. 12. Pl. 6.

Draw any chord A B, and bisect it by the perpendicular C D. Divide C D into two equal parts, and the point of bisection O will be the centre required.

Prob. 30.

To describe the circumference of a circle, through any three given points A, B, C, *provided they are not in a right line*, fig. 13. Pl. 6.

Draw the lines A B, B C, and bisect them by the perpendiculars D O, E O. From their intersection O, with the distance O A, O B, or O C, describe the circle A B C A, which will be the circumference required.

Prob. 31.

An arc A C *being given to complete the circumference*, fig. 14. Pl. 6.

Mark any three points A, B, C, on the given arc; join them by the right lines A B, B C, and proceed as in the preceding problem. The point O will be the centre, and O A, O B, or O C, will be the radius, for describing the circumference required.

Prob. 32.

To describe mechanically the circumference of a circle, through three given points A, B, C, *when the centre is inaccessible*, fig. 15. Pl. 6.

Place two rulers M N, R S, cross ways, touching the three points A, B, C. Fix them in V by a pin, and by a traverse piece T. Hold a pencil in A, and describe the arc B A C, by moving the angle R A N, so as to keep the outside edges of the rulers against the pins B C. Remove the instrument R V N, and on the arc described, mark two points D, E, so that their distance shall be equal

equal to the length B C. Apply the edges of the instrument against D, E, and with a pencil in G describe the arc B C, which will complete the circumference required.

Prob. 33.

To divide the periphery of any rectilinear plane figure A B C D E F, *into any number of equal parts; for instance, into seven,* fig. 16. Pl. 6.

Produce A B indefinitely both ways, and prolong B C to G, F E to H, and A F to I. Make C G equal to C D; B M to B G; E H to E F; F I to F H, and A L to A I. Divide L M into 7 equal parts. From B, with the radius B, 1, describe the arc 1, 1; from C with the radius C 1, the arc 1, 1; describe also from the points A, F, E, the arcs 4, 4; 5, 5; 6, 6. Then the intersections 1, 2, 3, 4, &c. made by the arcs upon the sides of the given figure, will be the points of division required.

Note. A plane figure having a re-entering angle, as fig. 17. Pl. 6. may likewise be divided by the same method.

Prob. 34.

To draw a right line equal to the circumference C D E F *of a given circle,* fig. 18. Pl. 6.

Method 1. The diameter of the circle being to its circumference, in proportion as 7 is to 22

 nearly.

nearly. Divide the diameter D F into 7 equal parts. Draw a line A B, on which set off 3 times the diameter D F plus $\frac{1}{7}$ of the same diameter, and the right line A B will be equal to the given circumference, as required.

Method 2. *By the sector*, fig. 18. Pl. 6.

Open the sector, and make the diameter D F, a transverse distance between 28 and 28, on the line of lines, marked L. Take the transverse distance between 88 and 88, which will be the right line required.

Note. Here the numbers 28 and 88, are multiples of 7 and 22, by 4.

Prob. 35.

To describe a circle which ſhall have its circumference equal to a given line D E, fig. 1. Pl. 7.

Divide D E into 22 equal parts, or its half into 11. Take $3\frac{1}{2}$ of these parts, which will be the radius for describing the circumference A B C, as required.

Prob. 36.

To find a right line equal to any given arc A B, fig. 2. Pl. 7.

First describe the circumference according to problem 31. Through the point A and the centre

tre O draw A D. Make C D equal to $\frac{1}{4}$ of the radius O C. Draw the indefinite line A E perpendicular to A D. Through D and B draw D E, and the line A E will be equal to the arc A B nearly.

Prob. 37.

To describe an ellipse (commonly called an oval) upon a given line A B, fig. 3. Pl. 7.

Divide A B into three equal parts. From the points C, D, describe the circles A E D I, B H C K. Through the intersection F and the centre C, draw F E; and through G and D the line G H. From F, with the radius F E, describe the arc E K; and from G, with radius G H, the arc H I, and A K B I will be the ellipse required.

Prob. 38.

To describe an ellipse, the length of its two diameters S *and* T *being given*, fig. 4. Pl. 7.

Method 1. Draw A B equal to S, and bisect it in G. Through G draw C D, perpendicular to A B: make G C, G D each equal to half the line T. From C, or D, with A G, or G B as radius, cut A B in E and F; and these points will be the foci of the ellipse. On E G, or F G, mark several points a, b, c, d, e, at any distance from each other. From E and F as centres, with the radius A a, describe arcs in H, I, K, L. Also from the

same centres E, F, and with the radius a B cut the arcs H, I, K, L; then from the same centres E, F with A b, B b describe arcs cutting each other in M, N, O, P, and so on with A c, B c; A d, B d, &c. Through the intersections H, M, I, N, O, K, describe a curve which will be the circumference of the ellipse required.

Note. The more points H, M, O, K are found, the easier the ellipse will be described by the hand.

Method 2, fig. 5. Pl. 7. Having drawn the two axes A B, C D perpendicular to each other, and equal to the given lines M, N, as in the preceding problem, the circumference of the ellipse may be described mechanically, as follows.

Take with a thread the length A G, or G B; and with this as a radius, and C as centre, cut A B in F and E. Take with the same thread the exact length A B; fix its ends by pins to the foci F, E, and move a pen, or a pencil, within the thread, so as to keep it always stretched, and it will describe the curve A C L B. Proceed in the same manner to describe the curve A H D B, and it will complete the ellipse required.

Prob. 39.

To find the greater and the less diameter, or the transverse and conjugate axes, of a given ellipse A B C D A, fig. 6. Pl. 7.

Draw

Draw any two parallel lines A N, H I; bisect each of them, and through the points of bisection L, M, draw P O. Bisect P O, and from the point of bisection E as a centre, and with any length as radius, describe the circle G F S, cutting the circumference of the ellipse, at four several points. Draw the chord F G, and through E draw C T parallel to F G; which will be the less diameter, or conjugate axis. Through E, draw D B perpendicular to C T, and D B will be the greater diameter, or transverse axis of the ellipse, as required.

Prob. 40.

To describe an equilateral triangle upon a given line A B, fig. 7. Pl. 7.

From the points A, B as centres, and with A B as radius, describe arcs intersecting each other in C. Draw C A, C B, and the figure A B C, will be the triangle required.

Note. An isosceles triangle, fig. T, may be formed in the same manner; taking for radius the given length A B of one of the equal sides.

Prob. 41.

To construct a triangle whose three sides shall be respectively equal to three given lines L, M, N; *provided any two of them be greater than the third,* fig. 8. Pl. 7.

Draw a line A B equal to L. From B as a centre and with M as radius, describe an arc a b. On A, with a radius equal to N, describe another arc cutting the former in C. Then draw the lines C A, C B, and A C B will be the triangle required.

Prob. 42.

To describe a square upon a given line A B, fig. 9. Pl. 7.

Method 1. From the point B, draw B C perpendicular, and equal to A B. On A and C, with the radius A B, describe arcs cutting each other in D. Draw the lines D A, D C, and the figure A B C D will be the square required.

Method 2, fig. 10. Pl. 7.

From A and B as centres, and with A B as radius, describe two indefinite arcs A C, B D, cutting each other in E. Bisect A E in F, and on E, with the radius E F, cross the two arcs in C and D. Draw A B, B C, D C, and A B C D will be the square required.

Prob. 43.

To describe a rectangle or parallelogram, whose length and breadth shall be equal to two given lines L *and* M, fig. 11. Pl. 7.

Draw A B equal to L, and make B C perpendicular thereto and equal to M. From the points C and

C and A, with the radii L and M, describe arcs intersecting in D. Join A D, D C, and A B C D will be the rectangle required.

Prob. 44.

To describe a regular pentagon on a given line A B, fig. 12. Pl. 7.

Method 1. Make B C perpendicular and equal to A B. Bisect A B in D, and from D as a centre, and with D C as radius, describe an arc C E cutting A B produced in E. With the centres A and B, and radius A E, describe arcs intersecting in F; then from F as a centre, and with A B as radius, cross those arcs in G and H. Join A G, B H, F G, F H, and they will complete the pentagon required.

Method 2, fig. 13. Pl. 7.

From the points A and B, with A B as a radius, describe two circles intersecting each other in C and D. Join C D, and from the intersection C, with the same radius A B, describe the arc L A B M, cutting the two circles in L and M, and the line C D in F. Draw the lines L F, M F, which produce to meet the circumference in H and E. From the points E and H, with the radius A B, describe arcs crossing each other in G, (or from the points A and B as centres, and with A H, or B E as radius, describe arcs cutting each other in G). Then

G). Then join A E, E G, G H, H B, and they will complete the pentagon required.

Prob. 45.

On a given line A B, *to describe a regular hexagon,* fig. 14. Pl. 7.

Upon A B, describe the equilateral triangle A B C. From C as a centre, and with C A, or C B as radius, describe the circle A B D E F G A. Set off the line A B round the circumference, from B to D, from D to E, &c. and join the points by lines, which will form the hexagon required.

Prob. 46.

To describe a regular octagon on a given line A B, fig. 15. Pl. 7.

Method 1. From the extremities A and B of the given line, erect the indefinite perpendiculars A D, B C. Produce A B both ways to K and L, and bisect the angles D A K, C B L, by the lines A E, B G. Make A E and B G each equal to A B. Through E and G draw the lines E F, G H parallel to A D, or B C, and each equal to A B. Make A D and B C each equal to E G, and join D F, D C, C H, and they will complete the octagon required.

Method

Method 2, fig. 1. Pl. 8.

Bisect A B in C. Draw C E perpendicular to A B. From C as a centre, and with C A as radius, describe the arc A D B. On D, with D A, or D B as radius, describe the arc A E B. Then the intersection E will be the centre, and E A, or E B the radius of a circle A F B, which will contain A B, the number of times required for an octagon.

Prob. 47.

To describe a regular nonagon on a given line A B, fig. 2. Pl. 8.

Bisect A B in C. Draw C F perpendicular to A B. From A as a centre, with A B as radius, describe the arc D B. Divide the arc D B into two equal parts in E. From D as a centre, with D E as a radius, describe the arc E F, and the point F will be nearly the centre of the nonagon required.

Prob. 48.

To describe a regular dodecagon on a given line A B, fig. 3. Pl. 8.

Bisect A B in C. Draw C D perpendicular to A B. From A, or B as a centre, and with the length A B cross C D in E, and from E, with E A as radius, describe the arc A D; then the point D will be the centre of the polygon required.

Prob.

Prob. 49.

To inscribe a square, or an octagon, in a given circle, fig. 4. Pl. 8.

For the square, or tetragon.

Draw the diameters A B, C D perpendicular to each other. Then draw the lines A D, A C, B D, B C, and A B C D will be the square required.

For the octagon.

Bisect any two arcs of the square A C, B C in G and E. Through the points G and E, and the centre O, draw lines which produce to F and H. Join A F, F D, D H, &c. and they will form the octagon required.

Prob. 50.

On a given line A B *to describe all the several polygons from the hexagon to the dodecagon inclusive*, fig. 5. Pl. 8.

Bisect A B by the perpendicular C D. From A as a centre, and with A B as a radius, describe the arc B E, which divide into six equal parts; and from E, as a centre, describe the arcs 5 F, 4 G, 3 H, &c. Then from the intersection E as a centre, and with E A as radius, describe the circle A I D B, which will contain A B six times. From F in like manner as a centre, and with F A

as

as radius, describe the circle A K L B, which will contain A B seven times; and so on for the other polygons.

Prob. 51.

To inscribe in a given circle an equilateral triangle, an hexagon, or a dodecagon, fig. 1. Pl. 9.

For the equilateral triangle, or trigon.

From any point D in the circumference, as a centre, and with the radius D O of the given circle, describe the arc A O B, cutting the circumference in A and B. Through D and O draw D C. Then join A B, A C, B C, and the figure A B C will be the triangle required.

For the hexagon.

Bisect the arcs A C, B C in E and F; and join A D, D B, B F, &c. which will form the hexagon. Or carry the radius D O six times round the circumference, and the required hexagon will likewise be obtained.

For the dodecagon.

Bisect the arc A D of the hexagon in G; and the line A G being carried twelve times round the circumference, will form the dodecagon required.

Prob. 52.

Another method to inscribe a dodecagon in a circle, or to divide the circumference of a given circle into

into 12 *equal parts, each of* 30 *degrees,* fig. 2. Pl. 9.

Draw the two diameters A B, C D perpendicular to each other. From the points A, C, B, D, as centres, and with A O as a radius, describe the arcs E O F, G O H, &c. and these arcs will, by intersecting the circumference, divide it into the required number of equal parts.

Prob. 53.

To inscribe a pentagon, an hexagon, or a decagon in a given circle, fig. 3. Pl. 9.

Draw the diameter A B, and make the radius D C perpendicular to A B. Bisect D B in E. From E as a centre, and with E C as radius, describe an arc cutting A D in F. Join C F, which will be the side of the pentagon; C D that of the hexagon, and D F that of the decagon.

Prob. 54.

To find the angles at the centre and circumference of a given polygon.

Divide 360 by the number of sides of the given polygon, and the quotient will be the angle at the centre, and this angle being subtracted from 180, the difference will be the angle at the circumference required. According to this method, the following table has been calculated, shewing the

the angles at the centres and circumferences, of regular polygons, from three to twelve sides inclusive.

Names.	Sides	Angles at the centre.		Angles at the circumference.	
Trigon	3	120°	0′	60°	0′
Tetragon	4	90	0	90	0
Pentagon	5	72	0	108	0
Hexagon	6	60	0	120	0
Heptagon	7	51	$25\frac{5}{7}$	128	$34\frac{2}{7}$
Octagon	8	45	0	135	0
Nonagon	9	40	0	140	0
Decagon	10	36	0	144	0
Undecagon	11	32	$43\frac{7}{11}$	147	$16\frac{4}{11}$
Dodecagon	12	30	0	150	0

Prob. 55.

To inscribe any regular polygon in a given circle, fig. 4. Pl. 9.

Method 1. From the centre C draw the radii C A, C B, making an angle equal to that at the centre of the proposed polygon, as contained in the preceding table. Then the distance A B will be one side of the polygon, which being carried round the circumference, the proper number of times, will complete the polygon required.

Method 2, fig. 5. Pl. 9.

Divide the diameter A B into as many equal parts, as the figure is to have sides. From A and

B as

B as centres, and with A B as radius, describe arcs intersecting each other at D. From D draw D C, through the second division of the diameter, and the line A C will be the side of the polygon nearly.

Method 3, fig. 6. Pl. 9.

Divide the diameter A B into as many equal parts, as the figure is to have sides. From the centre D, raise the perpendicular D E; and make C E equal to three-fourths of D C. Then from E, draw E F through the second division of the diameter, and the line A F will be the side of the required polygon, nearly.

Method 4, fig. 7. Pl. 9.

Draw the two radii C A, C B perpendicular to each other. Divide the quadrant A B into as many equal parts as the polygon is to have sides; then take four of them B D, which being carried round the circumference, will form the polygon required.

Note. The quadrant A B, fig. 8, may be readily divided into any required number of equal parts, by the quadratrix C D E, prob. 22.

Prob. 56.

To inscribe a circle in a given triangle A B C, fig. 9. Pl. 9.

Bisect

Bisect any two of the angles A and B, by the lines A D, B D, and from the point of intersection D, draw D F perpendicular to A B, and it will be the radius of the required circle.

Prob. 57.

To circumscribe a circle about any given triangle A B C, fig. 10. Pl. 9.

Bisect any two of the given sides A B, B C, with the perpendiculars E F, D F. From the intersection F as a centre, and with the distance of any of the angles, as a radius, describe a circle, and it will be the one required.

Prob. 58.

About a given square A B C D, *to circumscribe a circle*, fig. 11. Pl. 9.

Draw the two diagonals A C, B D intersecting each other in O. From the point of intersection O, as centre, and with O A, or O B, as radius, describe a circle, and it will be the one required.

Prob. 59.

About a given circle to circumscribe a square, fig. 12. Pl. 9.

Draw the two diameters A B, C D, perpendicular to each other. Through the points A, C, B, D,

draw the tangents E F, E G, G H, F H; and E G H F, will be the square required.

Prob. 60.

About a given circle to circumscribe a pentagon, fig. 13. Pl. 9.

First make the inscribed pentagon A B C D E (prob. 51). Bisect each side with the perpendiculars O I, O K, &c. and through the points A, B, C, D, E, draw the tangents H I, I K, K F, &c. Then the figure I K F G H, will be the circumscribed pentagon required.

Note. In the same manner any polygon may be circumscribed about a given circle.

Prob. 61.

To make a triangle similar and equal to a given triangle A B C, fig. 14 and 15. Pl. 9.

Draw a line D E, equal to A B. From the point D with A C, as radius, describe an arc in F, and from E with B C, as radius cut the former arc. Then draw the lines D F, E F; and D E F will be the triangle required.

Prob. 62.

To make a figure similar and equal to any given figure A B C D E, fig. 16 and 17. Pl. 9.

Divide

Divide the given figure into triangles, by the lines A D, A C, B E, B D. Draw a line F G equal to A B. On F G make the triangles F K G, F I G, F H G, equal and similar, to the triangles A E B, A D B, A C B, each to each (prob. 61). Then join I K, H I, and F G H I K will be the figure required.

SECT. II.

The REDUCTION and TRANSFORMATION of PLANE FIGURES.

Prob. 1.

TO make a triangle similar to a given triangle A B C, *one of its sides* D E *being given*, fig. 1 and 2. Pl. 10.

Make the angle D equal to the angle A, and the angle E, equal to the angle B; then the triangle D E F, will be similar to the triangle A B C.

Prob. 2.

Upon a given line A B *to describe a figure similar to a given figure* E G, fig. 3 and 4. Pl. 10.

Draw the diagonal E G, and upon A B make the triangle A B C, similar to the triangle E F G (prob. 1.) Then

1.) Then on A C make the triangle A C D, similar to the triangle E G H, and A B C D will be the figure required.

Prob. 3.

To make a figure similar to any given figure A C E; *one of its homologous sides being given,* fig. 5. Pl. 10.

Case 1. *When the figure is to be reduced according to the given side* M.

From any angle B, draw the diagonals B F, B E, B D, and on A B take D G equal to M. Then draw G L parallel to A F, and L K to F E, &c. and they will complete the figure required.

Case 2. *When the figure is to be enlarged, according to the given side* S, fig. 5. Pl. 10.

On B A produced, take B N equal to the given line S. Draw N O parallel to A F; and O P parallel to F E, &c. B N O P Q R will be the figure required.

Prob. 4.

To reduce a given figure A B C D E, *by means of a scale, one of the homologous sides* F G *of the required figure being given,* fig. 6 and 7. Pl. 10.

Divide the given figure into triangles, by the diagonals A C, A D, B E, B D. On the scale N belonging to the figure, measure A B, which suppose

pose to contain nine parts. Draw a line R S; on which take R T equal to F G. Divide R T into nine equal parts, and with these parts prolong the scale to any required length towards S. Then measure A C on the scale N. From F, with the same number of parts, taken on the scale R S, describe an arc at H. Find in the same manner the proportional length G H, and from G describe an arc cutting the former one in H. Join G H and F H, and proceed by the same method to describe the triangles F I G, F L G, similar to the triangles A D B, A E B. Then join the intersections L, I, H, by the lines F L, L I, I H, and they will complete the figure required.

Note. By the same method a figure may be enlarged, one of the homologous sides of the required figure being given. It may also be either reduced or enlarged, by describing angles at the points F and G, of the given side, equal to the corresponding angles of the triangles, in the given figure (prob. 1.)

Prob. 5.

To reduce a figure by the angle of reduction, or as it is sometimes called the angle of proportion, fig. 8, 9, and 10. Pl. 10.

Let A B be the given side on which it is required to describe a figure similar to F G H I K. Make any angle L M N, and on one of its sides

 M N,

M N, take M O equal to F G. From O, with the given length A B, cut M L in P. Join O P and draw several lines parallel to, and on both sides of it. Then draw the diagonals F I, F H, G K, G I. Take the length F K, and set it off from M towards N, on the side M N; and measure its corresponding line Q R, upon or between the parallels. From A with Q R describe an arc at E. Then take G K, and setting it off on M N, find its correspondent line S T. From B, with S T, cut the former arc in E, and join A E. Then proceed in the same manner to find the other points C and D, till the figure is completed.

Prob. 6.

To enlarge a figure A B C D E F G *by the angle of proportion, one of its homologous sides* H I *being given*, fig. 11, 12, and 13. Pl. 10.

Make any angle P Q R, as in the preceding problem. On Q R take Q S equal to A B. From S, with the given length H I, cut Q P in T. Join S T, and draw several parallel lines on both sides of it. Take the side A G and set it off from Q towards R, and measure its corresponding line U V. On H, with U V, describe an arc in O. Take in the same manner the correspondent line to B G; and on I, with X Y, cut the former arc at O. Join H O, and so on for the other sides.

Prob.

Prob. 7.

To reduce a map, or plan A B C D, *from one scale to another, by means of squares,* fig. 1 and 2. Pl. 11.

Divide the given figure A C by cross lines, forming as many squares, as may be thought necessary. Draw a line E F, on which set off as many parts from the given scale M, as A B contains parts of the scale N. Draw E H and F G perpendicular to E F, and each equal to the proportional parts contained in A D, or B C. Join H G and divide the figure E G into the same number of squares as the original A C. Describe in every square, what is contained in the correspondent square of the given figure, and E F G H will be the reduced plan required.

Note. The same operation will serve either to reduce or enlarge any map, plan, drawings, or paintings.

Prob. 8,

To make an isoscelis triangle equal to a given scalene triangle A B C, fig. 3. Pl. 11.

Bisect the base A B in E, and on A B erect the perpendicular E D. Draw C D parallel to A B, and from the intersection D, draw D A, D B, and A B D will be the required triangle.

Prob. 9.

To make an equilateral triangle equal to a given scalene triangle A B C, fig. 4. Pl. II.

On A B describe the equilateral triangle A B D. Produce D B towards E, and B A towards G, and draw C E parallel to A B. Bisect D E in I, and from I as a centre, and with I D as radius, describe the semicircle D F E. Draw B F, which is a mean proportional between B D, B E. From B as a centre, and with B F, as radius, describe the arc, F G H; and with the same radius, from G as centre intersect this arc at H. Then draw B H, G H, and B G H will be the triangle required.

Prob. 10.

To make a triangle equal to any given quadrilateral figure, A B C D, fig. 5. Pl. II.

Draw the diagonal A C, and make D E parallel thereto, intersecting B A produced in E. Then join E C, and E B C will be the triangle required.

Prob. 11.

To make a rectangle or a parallelogram equal to any given triangle, A B C, fig. 6. Pl. II.

Bisect the base A B in F, and through C draw C G parallel to A B. Draw F E, A D, parallel to each

each other, and either perpendicular to A B, or making any angle with it, as F D, A G. Then the rectangle A F E D, or the parallelogram A F D G, will be equal to the given triangle.

Prob. 12.

To make a rectangle equal to a given parallelogram, A B C D, fig. 7. Pl. II.

Produce A B towards F, and draw D E, C F, perpendicular to D C, or to A F; then the rectangle E F C D will be equal to the parallelogram A B C D.

Prob. 13.

To change a given triangle, A B C, *into another of an equal extent, but of a different height.*

Case I. *When the given point* D *is either in one of the sides, or in its prolongation,* fig. 8 and 9. Pl. II.

Draw a line from D to the opposite angle B, a and C E parallel to D B. a Then join D E, and A D E will be the triangle required.

Case 2. *When the point* D *is neither in one of the sides, nor in its prolongation,* fig. 10 and 11. Pl. II.

Draw the indefinite line A D F, and through C draw C F parallel to the base A B. Join F B, and the triangle A F B is equal to the triangle A B C; and the

the point D being in the same line with A F, proceed as in the first case, that is, join D B, and make F E parallel thereto; then join D E, and A D E will be the required triangle.

Case 3. *When the point* D *is nearly opposite and above the vertex,* fig. 1, *or within the triangle* A B C, fig. 2. Pl. 12.

From the point D draw D A, D B, and through the point C draw C E, C F parallel thereto. Then join D E, D F, and E D F will be the triangle required.

Prob. 14.

To make a rectangle equal to a given quadrilateral A B C D, fig. 3. Pl. 12.

Draw the diagonal D B, and parallel to it the lines A E, C F. Bisect B D by the perpendicular I H, and through the point D draw E F parallel to I H. Then E F H I will be the rectangle required.

Prob. 15.

To make a quadrilateral C D E F, *that ſhall be equal to the given pentagon* A B C D E, fig. 4. Pl. 12.

Produce E A towards F, and draw A C and B F parallel thereto. Join F C, and C D E F will be the quadrilateral required.

Prob.

Prob. 16.

To make a triangle equal to the given pentagon A B C D E, fig. 5. Pl. 12.

Produce A B both ways. Draw A D, and parallel to it, the line E F. Draw also B D, and parallel to it, the line C G. Then join D F and D G; and F D G will be the required triangle.

Prob. 17.

To make a triangle equal to a given polygon A B C D E, *having a re-entering angle* E, fig. 6. Pl. 12.

Produce A B towards F. Draw A D, and E G parallel thereto, and join D G. Draw also B D, and parallel to it, the line C F. Then join D F, and F D G will be the required triangle.

Prob. 18.

To make a triangle equal to the polygon A B C D E F *having one of its sides equal to* A F, fig. 7. Pl. 12.

Produce C D towards I, B C towards H, and A B towards G. Draw F D, and parallel to it, the line I H, and join F H, which will give a polygon A B H F, equal to the preceding one A B C I F, with one side less. Draw B F, and parallel to it, the line H G. Then join F G, and A F G will be the triangle required.

Prob.

Prob. 19.

To change a given polygon A B C D E, *into a triangle, its height* I H, *being given*, fig. 8. Pl. 12.

Produce A B both ways, and reduce the polygon to a triangle F D G, as has been shewn, problem 16. Draw F H, and parallel to it the line D L; draw likewise G H, and its parallel D M. Then join H L, H M, and L M H will be the required triangle.

Prob. 20.

To change any regular polygon A B C D E, *into a triangle, whose height shall be equal to* L M, *drawn from the centre* L *of the polygon, perpendicular to one of its sides*, A B, fig. 9. Pl. 12.

Produce A B both ways, on which set off the length A B or B C, as many times as the polygon has sides. From the centre L, draw L F, L G, and F L G will be the triangle required.

Prob. 21.

To change a rectangle A B C D, *into another, that shall be equal to it, and of a given length*, A E, fig. 10. Pl. 12.

Draw E F parallel to B C. Produce D C to F, and draw A F. Through the intersection G, draw H I parallel

H I parallel to A E, and A E I H will be the rectangle required.

Prob. 22.

To change a rectangle K L M N, *into another that shall be equal to it and of a given breadth,* K E, fig. 11. Pl. 12.

Draw E R parallel to K L, and produce N M towards B, and K L towards D. Through the intersection C, draw the diagonal K B, and make B D parallel to M L; then K D R E will be the required rectangle.

Prob. 23.

To describe a square, that shall be equal to a given rectangle, A B C D, fig. 12. Pl. 12.

Produce B A towards E, and A D towards F; and take A E equal to A D. Bisect E B in G, on which, as a centre, and with G E, or G B, as radius, describe the semicircle E F B. Then upon A F, describe the square A F K I, and it will be equal to the rectangle A B C D, as required.

Note. As any polygon may be changed into a triangle, by problem 16, sect. 2, and a triangle into a rectangle, by problem 11, sect. 2; a square may thus be described that shall be equal to any given polygon.

Prob.

Prob. 24.

To describe a square, that shall be equal to a given parallelogram, E F G H, fig. 13. Pl. 12.

Produce E F towards B. Draw F I perpendicular to E F, and take F B equal to F I. Bisect E B in C, on which, as a centre, and with C E, or C B, as radius, describe the semicircle E L B. Produce the perpendicular F I to L, and F L will be one of the sides of the required square, F L M N.

Prob. 25.

To change a square A B C H, *into a rectangle, one of its greater sides* M, *being given,* fig. 14. Pl. 12.

Produce A B, both ways, towards D and L; and take B D equal to M. Join C D, and bisect it by the perpendicular F O. From the intersection O, as a centre, and with O D, as radius, describe the semicircle D C L. On C B produced, take B E equal to B L. Draw E G parallel to B D, and D G parallel to B E, and D B E G will be the rectangle required.

Prob. 26.

To change a square E F G H, *into a rectangle, one of its less sides* N, *being given,* fig. 15. Pl. 12.

Produce

Produce E F, both ways, towards D and L. Take F L equal to N. Join L G, and bisect it by the perpendicular I O. From the point of intersection O, as a centre, and with O D, or O L, as radius, describe the semicircle L G D. On G F produced, take F B equal to F L, or to the given line N. Draw D C parallel to F B, and B C to F D, and they will form the required rectangle B C D F.

Prob. 27.

To describe any regular polygon, that shall be equal to a given triangle, fig. 1 and 2. Pl. 13.

Let it be required to describe a regular hexagon, equal to the given triangle A B C. First describe a regular hexagon, fig. 2, of any magnitude. On A B describe the triangle A B E, similar to the triangle D, the angle A E B being equal to the angle at the centre of the given polygon. Produce E B towards G. Draw C F parallel to A B, and join A F. And the triangle A B F is equal to the given triangle A B C. Divide B F into as many equal parts as the polygon is to have sides, which in this case is six. Take B G equal to B H, the $\frac{1}{6}$ of B F. Find B M, the mean proportional between B E and B G, by problem 14, sect. I. On B and with B M, as a radius, describe the arc M N. From the intersection N, as a centre, and with

with N B, as radius, describe the circle B O R B, in which inscribe an hexagon, which will be equal to the given triangle A B C.

Note. By this method any regular polygon may be described, that shall be equal to any irregular one, by changing first the irregular polygon into a triangle, as has been shewn, problem 16 and 17, sect. 2.

Prob. 28.

To describe a polygon that shall be equal to a given triangle, A B C, *and similar to a given polygon,* F G H D E, fig. 3 and 4. Pl. 13.

Draw the diagonal F H. Upon A B describe the triangle A B L, similar to the triangle F G H. Draw C K parallel to A B. Change the polygon F G H D E, into a triangle G H I, by problem 16 and 17, sect. 2. From B K cut off B M, in the same proportion as G F is to G I, by problem 17, sect. 1. Make B O equal to a mean proportional between B L and B M, by problem 15, sect. 1. Draw O P parallel to A L, and the triangle O B P will be similar to the triangle A B L, as also to the triangle G H F. On O P describe a quadrilateral P O R Q, similar to the quadrilateral F H D E, by problem 2, sect. 2, and P B O R Q will be the polygon required.

Prob.

Prob. 29.

To make a triangle equal to a given circle A B D A, fig. 5. Pl. 13.

Draw any radius C A, and make A E perpendicular to it, and equal to the circumference, by problem 34, sect. 1. Then join C E, and A C E will be the triangle required.

Prob. 30.

To describe a square that shall be equal to a given circle A F B I, fig. 6. Pl. 13.

Draw the diameter A B, and at its extremity B, draw the tangent B E, equal to the radius B C. On B E produced, take E G equal to the $\frac{1}{22}$ part of the radius C B. Join A G, and A I will be one of the sides of the required square nearly.

Note. By this method any triangle may be made equal to a given circle. First, by changing the given circle into a square, and the square into any triangle, by prob. 10, 8, 9 and 13, sect. 2.

Prob. 31.

To describe a circle that shall be equal to a given square A B C D, fig. 7. Pl. 13.

Bisect any side of the square B C in E. Take E F equal to $\frac{1}{12}$ part of B E. Join A F, on which,

as a diameter, describe the circle A B H, which will be equal to the given square nearly.

Note. By this problem, a circle may be made equal to any polygon, or triangle, by changing first the given polygon into a triangle, according to problem 16 and 17, sect. 2; then the triangle into a square, by problem 11 and 23, sect. 2.

Prob. 32.

On a given line A B, *to describe an ellipse that shall be equal to a given circle* C F D G C, fig. 8. Pl. 13.

Bisect A B in E, by the perpendicular D C. On E, as a centre, and with the radius of the given circle, describe the circumference D G C F. Draw B C, and bisect it in H, by the perpendicular H I. From the intersection I, as a centre, and with I B, as radius, describe the semicircle B C K. Take E N, E L, each equal to E K. Then the lines A B and L N are the two diameters of the required ellipse; for the construction of which, see problem 38, sect. 1.

Prob. 33.

To describe a circle that shall be equal to a given ellipse A B C D A, fig. 9. Pl. 13.

Draw

Draw the two diameters A C, B D perpendicular to each other, by problem 39, sect. 1. Make E F a mean proportional between the semidiameters E C, E D. From the point E, as a centre, and with E F, as radius, describe the required circle F G H I F, which will be the one required.

SECT. III.

The ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION of PLANE FIGURES.

ADDITION of PLANE FIGURES.

Prob. 1.

TO make a triangle that shall be equal to any number of triangles, when they are all of the same height, fig. 1. Pl. 14.

If for instance it be required to make a triangle equal to the three given triangles A B C, C D E, E F D. Draw a line A G equal to the sum of their bases, and join B G; and A B G will be the triangle required.

Note 1. When the triangles are of different heights, they must first be reduced to the same height, by problem 13, sect. 2, and then they may be added together, as above.

Note 2. When different polygons are required to be added together, they must likewise be first reduced into triangles of the same height.

Prob. 2.

To make a square that shall be equal to the sum of any given number of squares L, M, N, O, fig. 2, 3, 4, 5 and 6. Pl. 14.

Draw a line A B equal to one of the sides of the square L. On B erect the perpendicular B C, equal to one of the sides of the square M. Join A C, on which a square being constructed, will be equal to the sum of the two squares L and M. From the point C, draw C D perpendicular to A C, and equal to one of the sides of the square N. Draw the line A D, which will be the side of a square equal to the three squares L, M, N. On D, draw D E perpendicular to A D, and equal to one of the sides of the square O. Join A E, on which describe the square A E F G, which will be equal to the sum of the given squares L, M, N, O.

Prob. 3.

To describe a circle that shall be equal to the sum of any given number of circles N, O, P. fig. 7, 8, 9 and 10. Pl. 14.

Draw A B, B C perpendicular to each other. Take B A equal to the diameter of the circle N,

and

and B C equal to the diameter of the circle O. Draw A C, which will be the diameter of a circle equal to the sum of the two circles N and O. Draw C D perpendicular to A C, and equal to the diameter of the circle P. Then join A D, and it will be the diameter of the required circle.

Prob. 4.

To describe a figure that shall be equal and similar to any number of regular polygons E, F, G. fig. 11, 12, 13 and 14. Pl. 14.

Form a right angle A B C. Take B A equal to H I, and B C equal to K L. Draw A C, and perpendicular to it, the line C D equal to M N. Join A D, upon which constitute the polygon P, as required; see problem 44, sect. 1.

Prob. 5.

To describe a figure that shall be similar and equal to the sum of any given number of similar figures H, I, K. fig. 15, 16, 17 and 18. Pl. 14.

Draw lines A B, B C perpendicular to each other, or forming a right angle in B. Take B A equal to L M, and B C equal to the homologous side N O, and join A C. Draw C D perpendicular to A C, and equal to the homologous side P R. Join A D, on which describe a figure A D E F G, similar to one of the given figures, by problem

3, sect. 2, which will contain the sum of the three given figures, as required.

Note. Several figures, as 19, 20, 21, 22 and 23, may be added together, by reducing them first into triangles of the same height, by problem 13, sect. 2, and these triangles being added together, may also be changed into any other figure, as may be seen by problem 8, 9, 11, 27 and 28, sect. 2.

SUBTRACTION OF PLANE FIGURES.

Prob. 6.

To take from the triangle D E F *the triangle* A B C, *or to find their difference, when both are of the same height*, fig. 24. Pl. 14.

Cut off from the base D E the part E G, equal to the base A B. Draw the line G F, and the triangle D G F will be the difference required.

Note 1. If two triangles are not of the same height, they must be reduced to it, by problem 13, sect. 2, and then the difference may be found as above.

Note 2. When a polygon is to be deducted from another, and a triangle found equal to their difference; it may easily be effected by reducing them into triangles of the same height.

Prob.

Prob. 7.

To describe a square that shall be equal to the difference of two given squares, A C *and* E G, fig. 25 and 26. Pl. 14.

On the side D C of the greater square, describe the semicircle D H C. Take C H equal to the side E F of the less square. Draw the line D H, on which construct the square D I, which will be the difference required.

Note. The difference between any two given similar figures, or between two circles, may be obtained in the same manner.

MULTIPLICATION OF PLANE FIGURES.

Prob. 8.

To make a triangle that shall be equal to any multiple of a given triangle A B C, fig. 1. Pl. 15.

As for instance, let it be required to describe a triangle that shall be quadruple, the given triangle A B C. On A B produced, set off from A to E, four times the base A B. Draw the line C E, and A C E will be the triangle required.

Prob. 9.

To describe a square that shall be equal to any multiple of a given square A B C D, fig. 2. Pl. 15.

Draw the diagonal B D. Produce A B towards K, and A D towards G, on which take A H and A E, each equal to the diagonal B D. Upon A H construct the square A L; which will be equal to twice the square A C. Draw the line B E, which set off from A to I and from A to F, and the square described upon A I, will be equal to three times the square A C. Proceed in the same manner, by taking the line B F, upon which, construct a square which will be equal to four times the given square A C, and so on, if required.

Prob. 10.

To make a plan, or map, as many times as may be required, larger than a given one, E G. fig. 1 and 2. Pl. 11.

Suppose it be required to make it three times larger. After having reduced the given map, E G, into squares, by prob. 7, section 2; take one of the squares, E I, and find its triple by the preceding problem. Draw two indefinite lines, A B, A D perpendicular to each other. From A to B, set off the length of the side of the square found, the same number of times, as E K is contained in E F. In like manner set off the same length from A to D, as many times as there are divisions in E H. Through the several divisions

1, 2,

1, 2, 3, 4, &c. draw lines parallel to A B, and to A D. Then describe in every square of A C, what is contained in the correspondent square of the given figure, which will complete what was required.

Prob. 11.

To describe a polygon, that shall be similar and equal to any multiple of the given polygon, A B C D E, fig 3. Pl. 15.

Produce A B, A E indefinitely, as also the diagonals A C, A D. From the point B, draw B F perpendicular and equal to A B. From A, as a centre, and with A F, as radius, describe the arc F G. Upon A G, describe the similar polygon A G H I K, by problem 3, sect. 2, which will be equal to twice the given polygon. Draw F M parallel to A N, on which take F L equal to B G. From A, as a centre, and with A L, as radius, describe the arc L N. On A N describe a polygon in the same manner as before, which will be equal to three times the given polygon, and so on for any required number of times.

Prob. 12.

To describe a circle, that shall be equal to any multiple of a given circle A B D A, fig. 4. Pl. 15.

Draw

Draw the radii O B, O D perpendicular to each other. Produce O D indefinitely towards G. Take O E equal to B D. From the centre O, and with O E, as radius, describe the circle E H I E, which will be equal to twice the given circle A B D A. Take O F equal to B E. From O, as a centre, and with O F, as a radius, describe the circle F M N F, which will be equal to three times the given circle A B D A, and so on for any required number of times.

DIVISION OF PLANE FIGURES.

Prob. 13.

To divide a given triangle A B C, *into any number of equal parts, by lines drawn from the angle* C, fig. 5. Pl. 15.

Let it, for example, be required to divide the triangle A B C, into four equal parts. Divide the base A B, into four equal parts. Draw lines from C to the points of division D, E, F, and the triangle A B C will be divided as required.

Prob. 14.

To divide a given triangle A B C, *into four equal parts, by lines drawn from a point* D, *taken in one of its sides,* fig. 6. Pl. 15.

Reduce

Reduce the triangle A B C, into another A D E, by problem 13, sect. 2. Divide this triangle into four equal parts, as in the preceding problem. Join D C, and through the point F, draw F G parallel thereto. From D, draw lines to the divisions H, I and the intersection G, and they will divide the given triangle as required.

Prob. 15.

To divide the quadrilateral A B C D, *into two equal parts, by a line drawn from the angle* D, fig. 7. Pl. 15.

First change the quadrilateral into a triangle, A E D, by problem 10, sect. 2. Then divide the base, A E, into two equal parts in F. Draw D F, which will divide the quadrilateral as required.

Prob. 16.

To divide the quadrilateral A B E F, *into two equal parts, by a line drawn from the angle* E, fig. 8. Pl. 15.

Change the quadrilateral into a triangle B C E. Divide the triangle into two parts, by the line D E. Draw D G parallel to A E. Then join E G, and it will divide the figure as required.

Prob.

Prob. 17.

To divide the given pentagon A B C D E, *into three equal parts, by lines drawn from the angle* D, fig. 9. Pl. 15.

Reduce the given pentagon into a triangle, F D G, by problem 16, sect. 2. Divide the base F G, into three equal parts, at the points H, I. From the angle D, draw the lines D H, D I, and they will divide the pentagon as required.

Prob. 18.

To divide the given pentagon A B C D E, *into four equal parts, by lines drawn from the angle* B, fig. 10. Pl. 15.

Change the given pentagon into a triangle, A B F. Divide the base A F, into four equal parts, by the intersections G, H, I. Draw B E, and parallel to it the lines H K, I L. From the point B, draw lines to the intersections G, K, L, and they will divide the pentagon as required.

Prob. 19.

To divide any regular polygon into a given number of equal parts, by lines drawn from its centre.

Suppose it be required to divide the regular pentagon A B C D E, fig. 11. Pl. 15, into three equal

equal parts, by lines drawn from the centre O. Divide the periphery into three equal parts, by the intersections F, D, G, according to problem 33, sect. I. From the centre O, draw lines to the points D, G, F, and they will divide the pentagon, as required.

Another Method, fig. 12. Pl. 15.

Divide each of the sides of the pentagon, GHIKL, into three equal parts. From these divisions draw lines to the centre O, which will divide the pentagon into 15 equal triangles, and a line drawn at every fifth triangle, will give the required division of the pentagon.

Note. According to this method, any regular polygon may be reduced into any number of equal parts; by dividing each side of the given polygon, into the same required number of equal parts: and then drawing lines from its centre, to such number of divisions, as the given polygon has sides, which will complete what is required.

Prob. 20.

To divide a given polygon ABCDEF, *into any number of equal parts, by lines drawn to one of its angles* F, fig. 13. Pl. 15.

Let it be required to divide the given polygon into four equal parts. Change the given figure into

into a triangle A F G, by problem 16 and 18, sect. 2. Divide the base A G into four equal parts, at the points 1, 2, 3. Produce B C towards H. Draw F B, and through the divisions 2 and 3, draw the lines 2 I, 3 H, parallel to B F. Join F C, and draw H L parallel thereto. From the points of intersection 1, I, L, draw lines to the point F, which will divide the polygon as required.

Prob. 21.

To divide a given polygon A B C D E F, *into four equal parts, by lines drawn from a point* G, *taken in one of its sides* A F, fig. 1. Pl. 16.

Change the given polygon into a triangle A G H, whose vertex shall be at G. Divide the base A H, into four equal parts, at the points O, P, Q. Produce B C and C D indefinitely. Draw B G, and parallel to it the lines O I, P R, Q S. Join C G, and through the intersections R, S, draw the lines R L, S M, parallel thereto. Join D G, and through M, draw M N parallel to it. From the point G, draw lines to the intersections I, L, N, which will divide the polygon as required.

Prob. 22.

To make a square equal to ½, ⅓, ¼, &c. *of a given square* A B C D, fig. 2. Pl. 16.

Bisect

Bisect one of the sides D C, of the given square. From the point of bisection H, as a centre, and with H D, or H C, as radius, describe the semicircle D E C. Draw H E perpendicular to D C, and join D E, which will be the side of a square, equal to half the given one A C. To obtain a $\frac{1}{3}$ of the square A C, make D I equal to one third of D C. From the point I, draw I F perpendicular to D C. Join D F, upon which a square being constructed, will be equal to one third of the given square A C. And in the same manner D G may be found, which will be the side of a square equal to one fourth of the given square A C.

Prob. 23.

To draw a map equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. *of the given original* A C, fig. 1 and 2. Pl. 11.

Divide the given map A C, into squares, as has been shewn problem 7, sect. 2. Take one of the squares A P, and find its half, as has been shewn in the preceding problem (if the required reduction is to be one half the original). Draw two indefinite lines E F, E H perpendicular to each other; on which set off from E to F, as many times the side of half the given square, as there are divisions in A B. In like manner set off the same length from E to H, as many times

as

as there are divisions in A D. Through the several points 1, 2, 3, 4, &c. draw lines parallel to H G, and also to E H. Then describe in every square of E G, what is contained in the correspondent square of the given map A C, which will complete the required reduction.

Prob. 24.

To divide a regular polygon A B C A, *into any number of similar polygons,* fig. 3. Pl. 16.

Let it be required to divide the regular polygon A B C A, into six similar polygons. Upon any side C B, describe the semicircle C D B. Cut off B E equal to one sixth of B C. Draw E D perpendicular to B C. Join B D, on which describe a regular hexagon L, by problem 45, sect. 1. which will be one of the six polygons required.

Prob. 25.

To divide an irregular polygon A C B, *into any number of similar polygons,* fig. 4. Pl. 16.

Let it for example be required to divide the given irregular polygon A C B, into three similar polygons. Upon any one of its sides, A B describe the semicircle A E B. Cut off A D equal to one third of A B. Draw D E perpendicular to A B. Join A E, and upon it, as an homologous side to

A B,

A B, describe the polygon L, similar to the given one A C B, by problem 3, 4 and 5, sect. 2, which will be one of the three polygons required.

Prob. 26.

To divide a given circle A B C D, *into any number of circles*, fig. 5. Pl. 16.

Suppose it be required to divide it into five circles. Cut off A E equal to one fifth of the diameter A C. Draw E D perpendicular to A C. Then join A D, on which as a diameter, describe the circle A E D F; and it will be one of the five circles required.

Prob. 27.

To make a square in any proportion to a given square A B C D; *for instance, as* 3 *to* 7, fig. 6. Pl. 16.

Upon one of the sides D C, of the given square, describe the semicircle D F C. Divide D C into seven equal parts. From the third division E, draw E F perpendicular to D C; then join F C, and upon it describe the square F G, which will be in the required proportion.

Note. In the same manner, a circle, or any polygon, may be described according to a given proportion.

Prob. 28.

To make a map in any proportion to a given one; for instance, as 3 to 5, fig. 7. Pl. 16.

The map being divided into squares, as has been shewn problem 7. sect. 2. Draw a line E F, equal to the side of one of the squares, upon which describe the semicircle E G F. Divide E F into five equal parts. At the third division H, raise the perpendicular H G and draw F G, which will be the side of the required square. Then proceed according to problem 23. sect. 3.

SECT. IV.

MENSURATION of SUPERFICIES.

Prob. 1.

TO find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboides.*

* Area is the superficial measure contained within the surface of any plane figure; and the surfaces are measured by squares; as square inches, square feet, square yards, &c. A square whose side is one inch, or one foot, or one yard, &c. is called the *measuring unit*, as m, fig. 1. Pl. 17. by which the area, or the surface of any figure is computed.

1. Required

1. Required the area of the square ABCD, whose side is 5 feet, fig. 1. Pl. 17.

Multiply AB by BC, or 5 by 5, and the product 25 will be the number of square feet contained in the given square.

2. Required the area of the rectangle ABCD, whose length AB is 9 feet, and its breadth AD 4 feet, fig. 2. Pl. 17.

Multiply 9 by 4, and the product 36 will be the number of square feet in the required surface.

3. Required the surface of the rhombus ABCD, whose length AB is 7 yards, and its perpendicular height FC 6 yards, fig. 3. Pl. 17.

Multiply 7 by 6, and the product 42 will be the number of square yards, contained in the given figure.

4. Required the area of the rhomboides EFGH, whose length EF is 30 feet, and its perpendicular height BH, or DG 12 feet, fig. 4. Pl. 17.

Multiply 30 by 12, and the product 360, will be the number of square feet of the required area.

Example.

How many saucissons of 15 feet long and 11 inches in diameter, are required to line the interior slope of the parapet of a mortar battery, whose length is 15 toises, or 90 feet, and its height 7 feet 4 inches.

The

The thickness, 11 inches, being contained 8 times in the height 7 feet 4 inches, or 88 inches, and the length 15 feet, 6 times in the length 15 toises, or 90 feet. Multiply 8 by 6, and the product 48 will be the number of saucissons required.

Prob. 2.

To find the area of any triangle A B C, *its base* A B, *and its perpendicular height* D C, *being given*, fig. 5 and 6. Pl. 17.

Rule.

Multiply the base A B by the perpendicular D C, and half the product will be the area.

What will be the area of the triangle A B C, whose base A B is 20 feet, and its perpendicular 14 feet?

$\frac{20 \times 14}{2}$ = 140 square feet = area required.

Prob. 3.

To find the area of a triangle A B C, *whose three sides are given*, fig. 7. Pl. 17.

Rule.

From half the sum of the three sides, subtract each side severally; multiply the half sum and the

the three remainders continually together, and the square root of the product will be the area required*.

Example.

What will be the area of the triangle A B C, the side of which A B is 50 feet, B C 40 feet, and A C 30 feet?

$\frac{50+40+30}{2} = 60 =$ half sum of the three sides.

$60 - 30 = 30 =$ first difference.

$60 - 40 = 20 =$ second difference.

$60 - 50 = 10 =$ third difference.

$30 \times 20 \times 10 \times 60 = 360000$; of which the square root is $600 =$ area required.

Prob. 4.

Any two sides of a right angled triangle A B C, *being given, to find the third side*, fig. 8. Pl. 17.

Case 1. *When the two sides* A B, B C *forming the right angle are given, to find the hypothenuse* A C.

Rule.

Take the square root of the sum of the two squares A B and B C, and it will give the side A C.

Case 2. *When the hypothenuse* A C, *and one of the perpendicular sides* A B, *or* B C *are given.*

* See Mr. Bonnycastle's Mensuration.

Rule.

From the square of the hypothenuse subtract the square of the given side, and the square root of the remainder will be the side required.

Example 1, by Case 1. fig. 8. Pl. 17.

What will be the length A C, of the interior slope of a rampart, whose perpendicular height B C, is 17 feet, and the base A B, of the slope 20 feet?

$20 \times 20 = 400$ square of A B.

$17 \times 17 = 289$ square of B C.

$400 + 289 = 689 =$ sum of the two squares, of which the square root is 26·24 feet $=$ the length A C.

Example 2, by Case 1. fig. 9. Pl. 17.

What will be the length of the ladders B C, to escalade the escarp of a rampart, whose perpendicular height A C, is 30 feet, and the footing A B, required for the ladders 10 feet?

$30 \times 30 = 900 =$ square A C.

$10 \times 10 = 100 =$ square A B.

$900 + 100 = 1000 =$ sum of the two squares, of which the square root is 31·6 feet $=$ the length B C.

Example 3, by Case 1. fig. 10. Pl. 17.

If in the attack of a place, there be given the depth B A, of the ditch at the counterscarp equal to 18 feet,

18 feet, and the horizontal length B C, from the top of the counterscarp to the foot of the glacis equal to 20 toises, or 120 feet; What will be the length A C, of the descent of the ditch?

$120 \times 120 = 14400 =$ square of B C.

$18 \times 18 = 324 =$ square of B A.

$14400 + 324 = 14724 =$ sum of the two squares, of which the square root is 121·3 feet = the length of the descent A C.

Example 4, by Case 2. fig. 11. Pl. 17.

The gallery D B A leading to the chamber of a mine A, forms a right angle at B; of which the length A B is 9 feet; and the effect of the powder being supposed to extend every way from the chamber A, at the distance of 25 feet; What length of the gallery B D is required to be stopped up, so as to resist the same as the rest of the ground?

$25 \times 25 = 625 =$ square of 25.

$9 \times 9 = 81 =$ square of A B.

$625 - 81 = 544$ of which the square root is 23·32 feet, for the required length B C to be stopped up.

Prob. 5.

The three sides of a triangle A B C, *being given to find the length of the perpendicular* D C, *drawn from any angle to its opposite side*, fig. 12. Pl. 17.

Rule.

Multiply the sum of the two sides A C + B C by their difference A C — B C, and divide the product by the side A B, add half this quotient to half the length of the side A B, which will give the greatest length A D, and subtract half the same quotient from half the side A B, which will give the least length B D. By this means the triangle A B C is divided into two right angled triangles A D C, B D C, in each of which two sides A C, A D, and B C, B D being given, the perpendicular will be obtained by case 2, of the preceding problem.

Example.

What will be the length of the perpendicular D C, of the triangle A B C; A B being 60 feet, A C 46 feet, and B C 40 feet.

$\frac{\overline{46 + 40} \times \overline{46 - 40}}{60} = 8 \cdot 6 =$ quotient, or the difference between the two segments A D — B D.

$\frac{8 \cdot 6}{2} = 4 \cdot 3 =$ half difference.

$\frac{60}{2} + 4 \cdot 3 = 34 \cdot 3 =$ A D.

$\frac{60}{2} - 4 \cdot 3 = 25 \cdot 7 =$ B D.

Then in the right angled triangle B D C, there is given the side B D $= 25 \cdot 7$ feet, and the hypothenuse B C $= 40$ feet, to find the perpendicular D C.

$40 \times 40 = 1600 =$ square B C.

$25{\cdot}7 \times 25{\cdot}7 = 660{\cdot}49 =$ square B D.

$1600 - 660{\cdot}49 = 939{\cdot}51$ of which the square root is $30{\cdot}65$ feet $=$ D C, the perpendicular required.

Note. When the three sides of an isosceles triangle are given, one of its equal sides may be considered as the hypothenuse, and half the base as the other side of a right angled triangle; in which case the perpendicular will be obtained by the preceding problem.

Prob. 6.

To find the area of a trapezium A B C D, fig. 13. Pl. 17.

Draw the diagonal A C, upon which let fall from its opposite angles B and D, the perpendiculars B F, D E. Find by measurement the diagonal A C, and the perpendiculars B F, D E; then multiply the sum of the perpendiculars, by the diagonal, and half the product will be the area required.

Example.

What will be the area of the trapezium, whose diagonal A C is 100 feet, the perpendicular B F 40 feet, and the perpendicular D E 30 feet?

$\frac{\overline{40 + 30} \times 100}{2} = 350$ square feet $=$ area required.

Prob.

Prob. 7.

To find the area of a trapezoid A B C D, fig. 14. Pl. 17.

Rule.

Multiply the sum of the parallel sides A B, D C by the perpendicular distance E C, and half the product will be the area.

Example 1.

What will be the area of the trapezoid A B C D, of which the parallel sides A B, D C are 120 feet and 90 feet, and the perpendicular distance E C 40 feet?

$\frac{\overline{120 + 90} \times 40}{2} = 4200$ square feet = area required.

Example 2. fig. 15.

How many square feet of sod are wanted to line the interior slope of a rampart, whose perpendicular height A B is 17 feet, its base A E 20 feet, its length B C at the top 216 feet, and the length D E at the foot 207?

$17 \times 17 = 189$ = square of A B.

$20 \times 20 = 400$ = square of A E.

$400 + 189 = 589$, of which the square root is $24 \cdot 26$ = B E, the perpendicular distance between the parallels D E, C B.

$\frac{\overline{216 + 207} \times 24.26}{2} = 15 \cdot 131$ square feet = the quantity of sod required.

Prob.

Prob. 8.

To find the area of any irregular figure A B C D E, &c. fig. 16. Pl. 17.

Rule.

Draw diagonals, dividing the figure into trapeziums and triangles; then having found the area of each by prob. 2 and 6, sect. 4, add them together, and the sum will be the area required.

Example.

What will be the area of the figure A B C D, &c. having given A C = 42 feet; B I = 44 feet; G H = 35 feet; C G = 54 feet; F K = 50 feet; C E = 47 feet; D L = 24 feet, and F M = 41 feet?

$\frac{\overline{44 + 35} \times 42}{2} = 1659 =$ area of the trapezium A B C G.

$\frac{\overline{24 + 41} \times 47}{2} = 1527 \cdot 5 =$ area of the trapezium C D E F.

$\frac{54 \times 50}{2} = 1350 =$ area of the triangle G C F.

$1659 + 1527 \cdot 5 + 1350 = 4536 \cdot 5$ square feet = area required.

Prob. 9.

To find the area of a figure A B C D E, *having a part bounded by a curve* A B C, fig. 17. Pl. 17.

Draw

Rule.

Draw a right line A C, joining the extremities of the curve A B C; then find the area of the trapezium A C D E, by prob. 6, sect. 4. To A C let fall as many perpendiculars F G, H I, &c. as the several windings of the curve require. Find their lengths, and divide their sum by the number of perpendiculars, and the quotient will be the mean breadth, which being multiplied by A C, will give the area of the part A B C A, to which the trapezium being added, will give the area of the required figure.

Example.

What will be the area of the figure A E D C B A, of which E C is 178 feet; A R 69; D P 83; A C 160; G F 15; I H 24; S B 28; K L 22, and N M 10 feet.

$\frac{\overline{69+83} \times 178}{2} = 13528$ square feet = area of the trapezium A C D E.

$\frac{15+24+28+22+10}{5} = 19 \cdot 8$ = the mean breadth.

$160 \times 19 \cdot 8 = 3168$ square feet = area of the part A C B A.

$13528 + 3168 = 16696$ square feet = the area required.

Note. The two preceding problems, are commonly made use of in land surveying, where, instead

instead of measuring by the foot, Gunter's chain is used, to find in a more ready manner the number of acres contained in a given field. But should there be no Gunter's chain at hand, the superficial content in feet may be divided by 43560, and the quotient will be the number of square acres.

Prob. 10.

To find the area of a regular polygon.

Rule 1.

Multiply the perimeter of the polygon, by the perpendicular drawn from the centre upon one of the sides, and half the product will be the area.

Rule 2.

Multiply the area of one of the triangles by the number of sides, and the product will be the area of the polygon.

Example.

What will be the area of the regular hexagon ABCDEF (fig. 18. Pl. 17.) whose side AB is 40 feet, and the perpendicular GH 34·64 feet?

$40 \times 6 = 240 =$ the perimeter.

$\frac{240 \times 34.64}{2} = 4156{\cdot}8$ square feet $=$ area required.

Prob.

Prob. 11.

The diameter of a circle being given to find the circumference, or the circumference being given to find the diameter, fig. 1. Pl. 18.

The diameter or the circumference of a circle is found, the one from the other, by one of the following rules.

Rule 1.

As 7 is to 22, so is the diameter to the circumference.

As 22 is to 7, so is the circumference to the diameter.

Rule 2.

As 113 is to 355, so is the diameter to the circumference.

As 355 is to 113, so is the circumference to the diameter.

Rule 3.

As 1 is to 3·1416, so is the diameter to the circumference.

As 3·1416 is to 1, so is the circumference to the diameter.

Example 1 *by Rule* 1.

What will be the circumference of a circle, whose diameter A C is 20 feet?

7 : 22

7 : 22 : : 20 : circumference.

$\frac{22 \times 20}{7} = 62{\cdot}857$ feet = circumference

Example 2 *by Rule* 2.

What will be the diameter A C of a circle, whose circumference is 36 inches?

355 : 113 : : 36 : diameter A C.

$\frac{113 \times 36}{355} = 11{\cdot}459$ inches = diameter A C.

Example 3 *by Rule* 3.

What will be the circumference of a circle, whose diameter A C is 12 feet?

1 : 3·1416 : : 12 : circumference.

$3{\cdot}1416 \times 12 = 37{\cdot}6992$ feet = circumference.

Prob. 12.

To find the length of an arc A B, *the circumference* A D B A, *or the diameter* D B *being given*, fig. 2. Pl. 18.

Rule 1.

Case 1. *When the circumference is given, make the following proportion, as* 360° *is to the number of degrees in the arc, so is the circumference to the length of the arc.*

Rule

Rule 2.

Case 2. *When the diameter is given, first find the circumference by prob.* 11, *sect.* 4; *and then the length of the arc as in case* 1.

Example.

The arc A B being 70 degrees, and the circumference A D B A 60 feet, What will be the length of the arc A B?

Then $360° : 70° :: 60 :$ arc A B.

$\frac{70 \times 60}{360} = 11{\cdot}0666$ feet $=$ arc A B.

Prob. 13.

To find the area of a circle, fig. 1. Pl. 18.

The area of a circle is obtained by one of the following rules.

Rule 1.

Multiply half the circumference by half the diameter, and the product will be the area.

Rule 2.

Multiply the circumference by $\frac{1}{4}$ of the diameter, or by $\frac{1}{2}$ the radius, and the product will be the area.

Rule 3.

Multiply the circumference by the diameter, and $\frac{1}{4}$ of the product will be the area.

Rule

Rule 4.

Multiply the square of the diameter by ·7854*, and the product will be the area.

Example 1, *by Rule* 1.

What will be the area of a circle, whose circumference ACBD is 55·5488 inches, and its diameter AB 18 inches?

$\frac{55 \cdot 5488}{2} = 27 \cdot 7744 =$ half the circumference.

$\frac{18}{2} = 9 =$ half the diameter.

$27 \cdot 7744 \times 9 = 249 \cdot 9696$ square inches $=$ area.

Example 2, *by Rule* 4.

What will be the area of a circle, whose diameter AB is 12 feet?

$12 \times 12 = 144 =$ square of the diameter AB.

$\cdot 7854 \times 144 = 113 \cdot 0976$ square feet $=$ area.

Prob. 14.

The area of a circle ACBDA, *being given to find the diameter* AB, fig. 1. Pl. 18.

Rule.

Divide the area of the circle by ·7854, and take the square root of the quotient, which will be the diameter.

* See Mr. Bonnycastle's Mensuration.

Example.

What will be the diameter A B of the circle A C B D, its area being 176·7150 square feet?

$\frac{176.7150}{.7854} = 225$, of which the square root is 15 feet = the diameter required.

Prob. 15.

To find the area of a semicircle A B C A, fig. 3. Pl. 18.

Rule 1.

Multiply half the semicircumference by the radius D A, and the product will be the area.

Rule 2.

Multiply the square of the diameter A C, by ·7854, and half the product will be the area.

Example, by Rule 2.

What will be the area of the semicircle A B C A, its diameter A C being 50 inches?

$50 \times 50 = 2500 =$ square A C.

$\frac{.7854 \times 2500}{2} = 981 \cdot 75$ square inches = area required.

Prob. 16.

To find the area of a sector, fig. 2. Pl. 18.

Rule.

Rule.

Multiply the radius C A by the arc A B, and half the product will be the area.

Example.

What will be the area of the sector A B C A, its radius B C being 30 inches, and the length of the arc A B, 36·6 inches?

$\frac{36.6 \times 30}{2} = 549$ square inches = area required.

Prob. 17.

To find the area of the segment of a circle, fig. 4. Pl. 18.

Rule

1. Find the area of the sector A D C B A, or that of A D C E F A, by the preceding problem, according as the area of the less or greater segment is required.

2. Find the area of the triangle A C D, formed by the chord A C of the segments, and the radii D A, D C of the sectors.

3. Then the sum or difference of these areas, according as the segment is greater, or less than a semicircle, will be the area.

Example 1.

What will be the area of the less segment A C B A, the radius D A being 20 inches, the chord

AC 22·42 inches, the length of the arc ABC 24·43 inches, and the perpendicular DG 16·56 inches?

$\frac{24.43 \times 20}{2}$ = 244·3 square inches = area of the sector ABCDA.

$\frac{22.42 \times 16.56}{2}$ = 185.6376 square inches = area of the triangle ADC.

244·3 — 185·6376 = 58·6624 square inches = area required.

Example 2.

What will be the area of the greater segment ACEFA, the length of the arc AFEC being 101·23 inches, the radius DA, the chord AC, and the perpendicular DG to be of the same dimensions as those given in the preceding example?

$\frac{101.23 \times 20}{2}$ = 1012·3 square inches = area of the sector ADCEF.

$\frac{22.43 \times 16.56}{2}$ = 185.6376 square inches = area of the triangle ADC.

1012·3 + 185·6376 = 1197·9376 square inches = area required.

Prob. 18.

To find the area of the space ABDEA, *included between two parallel chords* AB, ED, *and the two arcs* AE, BD, fig. 5. Pl. 18.

Rule.

Rule.

Find the area of each segment EFDE, and AFBA, and their difference will be the area required.

Example.

What will be the area of the space ABDEA, the radius CB, or CD being 20 inches, the length of the arc EFD 48·8693 inches, the length of the arc AFB 24·4346, the greater chord ED 37·6 inches, and the less chord AB 23·4 inches?

$\frac{48.8693 \times 20}{2} = 488{\cdot}693 =$ area of the sector EFDCE; (see prob. 16. sect. 4.)

$\frac{37.6 \times 6.8}{2} = 127{\cdot}84 =$ area of the triangle CED; (see prob. 4. sect. 4.)

$488{\cdot}693 - 127{\cdot}84 = 360{\cdot}853 =$ area of the segment EFDE.

$\frac{24.4346 \times 20}{2} = 244{\cdot}346 =$ area of the sector AFBCA.

$\frac{23.4 \times 16.6}{2} = 194{\cdot}22 =$ area of the triangle ABC.

$244{\cdot}346 - 194{\cdot}22 = 50{\cdot}126 =$ area of the segment AFBA.

Then $360{\cdot}853 - 50{\cdot}126 = 310{\cdot}727$ square inches $=$ area required.

Note. When the centre C of the circle is within the space ABEDA, as fig. 6, from the area of the circle subtract the sum of the areas of the two segments DFED, AGBA, and the difference will be the required area. And the same rule is also observed, whether the two chords are parallel, or otherwise, as in fig. 7. Pl. 18.

Prob. 19.

To find the area of a ring included between the two circumferences ABCDA, EFGHE *of two concentric circles*, fig. 8. Pl. 18.

Rule.

Multiply half the sum of the circumferences, by half the difference of their diameters, and the product will be the area.

Example.

What will be the area of the ring AFCHA, the diameter AC being 72 inches, and the diameter EG 40 inches?

$3{\cdot}1416 \times 72 = 226{\cdot}1952 =$ circumference ABCDA; (see prob. 11. sect. 4.)

$3{\cdot}1416 \times 40 = 125{\cdot}6640 =$ circumference EFGHE.

$\frac{226.1952 + 125.6640}{2} = 175{\cdot}9296 =$ half the sum of the two circumferences.

72 — 40

$\frac{72-40}{2} = 16 =$ half difference of the two diameters A C, E G.

$175\cdot9296 \times 16 = 2814\cdot8736$ square inches = area required.

Note. In the same manner may be obtained, the area of any part A B F E A of the ring, included between the lines A E, B F, and the arcs A B, E F, by multiplying half the sum of the two arcs by A E, half the difference of the two diameters A C, E G, or by the difference of the two radii N A, N E.

Prob. 20.

To find the area of a lune, or the space A C B D A, *included between the intersecting arcs* A C B, A D B *of two excentric circles,* fig. 9. Pl. 18.

Rule.

To find the area of each segment A C B A, A D B A, by prob. 16. sect. 4, and their difference will be the required area of the lune A C B D A.

Prob. 21.

To find the area of an ellipse A M D L, *according to the construction of prob.* 31. *sect.* 1. fig. 10. Pl. 18.

Rule.

Find the sum of the areas of the sectors A F G B, F K E L, B I C M, and E G C D, by prob. 16. sect. 4,

from which subtract the area of the lozenge GIHK, and the difference will be the required area.

Prob. 22.

To find the area of an ellipse ACBD, fig. 11. Pl. 18.

Rule.

Multiply continually together the two diameters AB, CD, and the number 11. Divide the last product by 14, and the quotient will be the area nearly true.

Example.

What will be the area of the ellipse ADBCA, its transverse AB being 15 feet, and its conjugate CD 10 feet?

$\frac{11 \times 15 \times 10}{14} = 117 \cdot 85$ square feet = area required.

Another method still nearer.

Rule.

Multiply continually together the two diameters, and the number ·7854, and the product will be the area of the ellipse.

Example.

What will be the area of the ellipse ADBCA, its transverse AB being 25 inches, and conjugate CD 18 inches?

·7854

$\cdot 7854 \times 25 \times 18 = 353 \cdot 43$ square inches = area required.

Prob. 23.

To find the area of the parabola A B C A, fig. 12. Pl. 18.

Rule.

Multiply the base A C by the height D B, and the $\frac{2}{3}$ of the product will be the area.

Example.

What will be the area of the parabola A B C A, its base A C being 20 feet, and its height D B 12 feet?

$20 \times 12 = 240.$

$\frac{240 \times 2}{3} = 160$ square feet = area required.

SECT. V.

MENSURATION of SOLIDS.

DEFINITIONS.

1. A *Solid*, is a body contained under three dimensions, or extended in length, breadth and thickness.

2. Solids are measured by cubes, whose sides are each an inch, a foot, a yard, &c. and the solidity,

solidity, capacity, or content of any figure, is computed by the number of such cubes as are contained in it.

3. Solidities are terminated, either by one surface, as a globe, or by several surfaces, either plane or curved.

4. A *Cube*, is a solid contained by six equal square sides, as fig. 1. Pl. 19.

5. A *Parallelepipedon*, is a solid comprehended under six parallelograms, every opposite two of which are equal and parallel, as fig. 2. Pl. 19.

6. A *Prism*, is a solid, whose ends are two equal and similar plane figures, and its sides parallelograms, as fig. 3. Pl. 19.

It is called a *triangular prism*, when its ends A B E, G H F are triangles; a *square prism*, when its ends are squares; a *pentagonal prism*, when its ends are pentagons, and so on.

7. A *Cylinder*, is a solid described by the revolution of a right angled parallelogram C D E F, about one of its sides C D, which remains fixed, fig. 4. Pl. 19.

8. A *Pyramid*, is a solid whose sides are all triangles, meeting together in a point, and the base any plane figure whatever, as fig. 5. Pl. 19.

It is called a *triangular pyramid*, when its base is a triangle; a *square pyramid*, when its base is a square; a *pentagonal pyramid*, when its base is a pentagon, and so on.

9. A Prism

9. A Prism, or a pyramid, is regular, or irregular, according as its base is a regular, or irregular plane figure.

10. A *Cone*, is a round pyramid, of which the base is a circle, as fig. 6. Pl. 19.

11. A line C D drawn from the vertex to the centre of the base, or through the centres of the two ends, is called the *axis* of a solid, fig. 3, 4, 5, 6. Pl. 19.

12. When the axis C D is perpendicular to the base, it is a *right prism*, *pyramid*, or *cone*, otherwise it is oblique.

13. The *Segment of a pyramid*, *cone*, or any other solid, is a part D E F G, cut off from the top by a plane D E F, parallel to the base A B C, fig. 7. Pl. 19.

14. A *Frustum*, or *Trunk*, is the part A B C D E F, that remains at the bottom, after the segment is cut off, fig. 7.

15. An *Ungula*, or *Hoof*, is a part of a cylinder or cone, cut off by a plane, passing obliquely through the plane of the base, and one of the sides of the solid, as A B C D A, fig. 8. Pl. 19.

16. A *Sphere*, is a solid contained under one convex surface, and is described by the revolution of a semicircle about its diameter, which remains fixed, fig. 9. Pl. 19.

17. The *Centre of the sphere*, is such a point C within the solid, as is every where equally distant from the convex surface of it, fig. 9.

18. A *Diameter*

18. A *Diameter of a sphere*, is a straight line A B, which passes through the centre C, and is terminated both ways by the convex surface. This line is also called the *axis of the sphere*, fig. 9.

19. A Circle A E B F A, which divides the sphere into two equal parts, or *hemispheres*, is called a *great circle of the sphere*, fig. 9.

20. A Circle G H I K G, which divides the sphere into two unequal parts, is called a *less circle of the sphere*, fig. 9.

21. A *Segment of a sphere*, is a part D cut off by a plane, the section of which is always a circle G H I K G, called the *base of the segment*, fig. 9.

22. *A Sector of a sphere*, is that which is composed of a segment A D B F A G less than an hemisphere, and of a cone A E B G A F, fig. 10. Pl. 19.

23. A *Zone of a sphere*, is that part which is intercepted between two parallel planes A B D E A, F G H I F; and when these planes are equally distant from the centre C, it is called the *middle zone of the sphere*, fig. 11. Pl. 19.

24. A *Spheroid*, or as it may be more properly called an *Ellipsoid*, is a solid, generated by the revolution of a semi-ellipse, about one of its diameters, which remains fixed, fig. 12. Pl. 19.

There are two sorts of spheroids, *prolate* and *oblate*.

The

The spheroid or ellipsoid, is called prolate, when the revolution is made about the transverse diameter A B, and oblate when it is made about the conjugate diameter C D.

Prob. 1.

To find the surface of a cube A D, fig. 1. Pl. 20.

Rule.

Multiply the square of one of the linear sides A B by 6 (the number of faces of the cube), and the product will be the area.

Example.

Suppose the linear side A B to be 5 inches, what will be the area of the cube?

$5 \times 5 = 25 =$ square of A B.

$25 \times 6 = 150$ square inches $=$ area required.

Prob. 2.

To find the solidity of a cube A D, fig. 2. Pl. 20.

Rule.

Multiply the square of the linear side A B, by the side A C, or A B, and the product will be the solidity.

Example.

What will be the solidity of the cube A D, whose linear side A B, or A C is 7 inches?

7×7

$7 \times 7 = 49 =$ square of A B.

$49 \times 7 = 343$ cubic inches $=$ the solidity required.

Prob. 3.

To find the solidity of a parallelepipedon E B, fig. 3. Pl. 20.

Rule.

Multiply the length A B by the breadth A C, and that product again by the thickness, or depth C E, and it will give the solidity required.

Example.

What will be the solidity of the parallelepipedon E B, whose length A B is 7 feet, its breadth A C 4 feet, and its thickness C E 3 feet?

$7 \times 4 \times 3 = 84$ solid feet $=$ solidity required.

Prob. 4.

To find the surface of a right prism B D C, fig. 4. Pl. 20.

Rule.

Multiply the perimeter of the base A B C A, by one of the linear edges A D, to which product add the areas of the two ends A B C, D E F, and their sum will be the whole surface.

Example.

Example.

What will be the surface of the prism A E C, whose linear side A B, or A C is 6 inches, its length A D 12 inches, and the perpendicular A G of the triangle C A B 5·19 inches?

$6 \times 3 = 18 =$ perimeter A B C A.

$18 \times 12 = 216 =$ area of the three faces.

$\frac{6 \times 5.19}{2} = 15·57 =$ area of the triangle A B C.

$15·57 \times 2 = 31·14 =$ area of the two ends.

$216 + 31·14 = 247·14$ square inches $=$ area required.

Note 1. In the same manner may be obtained the surface of a right prism of any number of sides, whether its ends are regular, or irregular polygons.

Note 2. When it is required to find the surface of an oblique prism, fig. 5. the surface of its sides and ends must be calculated separately, and their sum will be the whole surface.

Prob. 5.

To find the solidity of a right prism, fig. 6. Pl. 20.

Rule.

Multiply the area of the base A B C D E by the height, or length A F, and the product will be the solidity.

Example.

$7 \times 7 = 49 =$ square of A B.

$49 \times 7 = 343$ cubic inches $=$ the solidity required.

Prob. 3.

To find the solidity of a parallelepipedon E B, fig. 3. Pl. 20.

Rule.

Multiply the length A B by the breadth A C, and that product again by the thickness, or depth C E, and it will give the solidity required.

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To find the surface of a right prism B D C, fig. 4. Pl. 20.

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Multiply the perimeter of the base A B C A, by one of the linear edges A D, to which product add the areas of the two ends A B C, D E F, and their sum will be the whole surface.

Example.

Example.

What will be the surface of the prism A E C, whose linear side A B, or A C is 6 inches, its length A D 12 inches, and the perpendicular A G of the triangle C A B 5·19 inches?

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$\frac{6 \times 5.19}{2} = 15 \cdot 57 =$ area of the triangle A B C.

$15 \cdot 57 \times 2 = 31 \cdot 14 =$ area of the two ends.

$216 + 31 \cdot 14 = 247 \cdot 14$ square inches $=$ area required.

Note 1. In the same manner may be obtained the surface of a right prism of any number of sides, whether its ends are regular, or irregular polygons.

Note 2. When it is required to find the surface of an oblique prism, fig. 5. the surface of its sides and ends must be calculated separately, and their sum will be the whole surface.

Prob. 5.

To find the solidity of a right prism, fig. 6. Pl. 20.

Rule.

Multiply the area of the base A B C D E by the height, or length A F, and the product will be the solidity.

Example.

Example.

Required the solidity of the pentagonal prism A G, whose linear side A B or B C at the base, is 8 inches, the perpendicular I K 5·5 inches, and the length A F 24 inches?

$8 \times 5 = 40 =$ perimeter A B C D E A.

$\frac{40 \times 5.5}{2} = 110.0 =$ area of the base A B C D E A (see prob. 10. sect. 4.)

$110.0 \times 24 = 2640.0$ cubic inches $=$ solidity required.

Prob. 6.

To find the solidity of a quadrangular prism D E, *whose base is a trapezium* A B C D, fig. 7. Pl. 20.

Rule.

Multiply the area of the trapezium A B C D (see prob. 6. sect. 4.) by the length A F, and the product will be the solidity.

Example 1.

What will be the solidity of a bank of earth D E, whose length A F is 250 feet, the diagonal A C 27 feet, the perpendicular D H 15 feet, and the perpendicular B G 6 feet?

$\frac{\overline{15 + 6} \times 27}{2} = 183.5$ square feet $=$ area of the trapezium A B C D.

183·5

$183 \cdot 5 \times 250 = 45875$ cubic feet = solidity required.

Example 2.

Required the solidity of the revetement A B of a rampart, (fig. 8.) whose thickness D E at the top is 5 feet, the base A C 11 feet, the height A D 36 feet, and the length E B 120 feet?

$\frac{\overline{11+5} \times 36}{2} = 288$ = area of the trapezoid A C E D (see prob. 7. sect. 4.)

$288 \times 120 = 34560$ cubic feet = solidity required.

Example 3.

It is required to find the solidity of the rampart A B D K L, its parapet and banquette included, fig. 9. Pl. 20?

Divide the profile A F B into a trapezium, trapezoids and triangles, by prob. 8. sect. 4. in which suppose to be given A B = 87 feet; E C = 57 feet; M C = 18 feet; E F = 21·5 feet; H P = 7 feet; D R = 3 feet; H I = 9 feet; N O = 4 feet; H N = 3 feet; and B L = 200 feet.

$\frac{\overline{87+57} \times 18}{2} = 1296$ = area of the trapezoid A B C E.

$\frac{\overline{7+3} \times 21.5}{2} = 107 \cdot 5$ = area of the trapezium E H F D.

$\frac{\overline{9+4}\times 3}{2} = 19\cdot 5 =$ area of the trapezoid HNOI.

$1296 + 107\cdot 5 + 19\cdot 5 = 1423\cdot 0 =$ area of the profile AFB.

$1423\cdot 0 \times 200 = 284600$ cubic feet = solidity required.

Prob. 7.

To find the breadth of a ditch, whose length and depth are given, having a slope at the escarp and counterscarp, each equal to half the depth of the ditch, in order to produce a required number of solid feet of earth, to construct the parapet of a mortar battery, fig. 10. Pl. 20.

Rule.

Divide the given content by the length of the ditch, and the quotient again by the depth; then to this last quotient add half the depth, and the sum will be the required breadth.

Example.

What will be the breadth AB of a ditch, whose length BE is 60 feet, and depth DC 6 feet, having a slope on each side of 3 feet; 6480 solid feet of earth being required to construct the parapet FG?

$\frac{6480}{60} = 108 =$ first quotient.

108

$\frac{108}{6} = 18 =$ second quotient.

Then $18 + 3 = 21$ feet $=$ A B, the required breadth.

Prob. 8.

To find the convex surface of a cylinder A F, fig. 11. Pl. 20.

Rule.

Multiply the circumference A B C D A by the length A E, and the product will be the convex surface required.

Example.

What will be the convex surface of the cylinder A F whose diameter A C is 8 inches, and length A E 20 inches?

$3{\cdot}1416 \times 8 = 25{\cdot}1328 =$ circumference A B C D A.

$25{\cdot}1328 \times 20 = 502{\cdot}6560$ square inches $=$ surface required.

Note. To obtain the whole surface of the cylinder A F, add twice the area of one of its ends to the convex surface, and their sum will be the whole surface required.

Prob. 9.

To find the solidity of a cylinder A F, fig. 11. Pl. 20.

Rule.

Multiply the area of the base A B C D A by its length A E, and the product will be the solidity.

Example.

What will be the solidity of the cylinder A F, whose diameter A C is 20 inches, and length A E 40 inches?

$20 \times 20 = 400 =$ square of A C.

$\cdot7854 \times 400 — 314\cdot1600 =$ area A B C D A.

$314\cdot1600 \times 40 = 12566\cdot4$ cubic inches = solidity required.

Prob. 10.

To find the solidity of an oblique prism, or an oblique cylinder, fig. 12 and 13. Pl. 20.

Rule.

Multiply the area of one of the ends by the perpendicular C D, and the product will be the solidity.

Example.

What will be the solidity of the cylinder A D, fig. 13. Pl. 20, whose diameter A B is 10 inches, and its perpendicular height C D 25 inches?

$10 \times 10 = 100 =$ square A B.

$\cdot7854 \times 100 = 78\cdot5400 =$ area of the circle A E B F A.

78·5400

$78 \cdot 5400 \times 25 = 1963 \cdot 5$ cubic inches = solidity required.

Prob. 11.

The solidity and the length of a cylinder being given, to find the area of one of its ends A B C D, *and its diameter* A C, fig. 11. Pl. 20.

Rule.

Divide the content by the length, and the quotient will be the area of one of its ends; then dividing the area found by ·7854, the square root of this last quotient will be the diameter required.

Example.

What will be the area of one of the ends A B C D A, the solidity of the cylinder A F being 565·4880 cubic inches, and the height, or length A E 20 inches?

$\frac{565.4880}{20} = 28 \cdot 2744 =$ area of one of the ends A B C D.

$\frac{28.2744}{\cdot 7854} = 36$, of which the square root is 6 = the diameter required.

Prob. 12.

To find the content of the solid part of a hollow cylinder, fig. 14. Pl. 20.

Rule 1.

From the content of the cylinder A C, subtract the content of the cylinder G F, and the difference will be the solidity (see prob. 9.)

Rule 2.

Multiply the area of the ring D H F L (prob. 18. sect. 4) by the height A D, and the product will be the solidity.

Example, by Rule 1.

What will be the content of the solid part of the hollow cylinder A C, whose diameter A B is 12 inches, the diameter E F 8 inches, and the height A D 20 inches?

$12 \times 12 = 144 =$ square of the diameter A B.

$\cdot7854 \times 144 = 113\cdot0976 =$ area of the circle A B.

$113\cdot0976 \times 20 = 2261\cdot9520 =$ content of A C.

$8 \times 8 = 64 =$ square of E F.

$\cdot7854 \times 64 = 50\cdot2656 =$ area of the circle E F.

$50\cdot2656 \times 20 = 1005\cdot3120 =$ content of G F.

$2261\cdot9520 - 1005\cdot3120 = 1256\cdot64$ cubic inches $=$ solidity required.

Prob. 13.

To find the solidity of the frustum of a prism A D, fig. 1. Pl. 21.

Rule.

Rule.

Multiply the area of the base A B C, by the sum of the three edges A F, B E, C D, and $\frac{1}{3}$ of the product will be the solidity.

Example.

What will be the solidity of the frustum of the prism A D, whose three edges A F, B E, C D are 8, 9 and 12 feet, one of the sides A B 6 feet, and the perpendicular A G 5·19 feet?

$\frac{5.19 \times 6}{2} = 15.57 =$ area of the base A B C.

$8 + 9 + 12 = 29 =$ sum of the three edges.

$\frac{15.57 \times 29}{3} = 150.51$ cubic feet $=$ solidity required.

Prob. 14.

To find the solidity of a part A F D B *of any triangular prism, whose ends are neither parallel to each other, nor perpendicular to its sides,* fig. 2. Pl. 21.

Rule.

Multiply the area of the perpendicular section G H I, by the sum of the three edges A B, F C, E D, and $\frac{1}{3}$ of the product will be the solidity.

Example.

What will be the solidity of the triangular prism A F D, whose three edges A B, F C, E D are

5 feet 4 inches, 4 feet 2 inches, 2 feet 6 inches, GH 15 inches, HI 15 inches, and GI 10 inches?

Find the area of the perpendicular section GHI by prob. 3, or by the note of prob. 5. sect. 4, which will be 70·7 square inches.

5 feet 4 inches = 64 inches.

4 —— 2 ———— = 50 ————

2 —— 6 ———— = 30 ————

Then $\frac{\overline{64 + 50 + 30} \times 70.7}{3} = 3393{\cdot}6$, cubic inches = solidity required.

Note. In the same manner the solidity of a *cuneus*, or *wedge* may be obtained, fig. 3. Pl. 21.

Example.

What will be the solidity of the wedge ADE, whose edges AB, CD, EF are 9 inches, 9 inches and 7 inches; and the sides of the perpendicular section LMN, that is LM, MN each 14 inches, and the base LN 4 inches?

Find the area of the perpendicular section LMN, as has been shewn in the preceding problem, which will be 27·712 square inches.

Then $\frac{27.712 \times \overline{9 + 9 + 7}}{3} = 230{\cdot}933$ cubic inches = solidity.

Prob. 15.

To find the solidity of the frustum of a prism, of any number of sides, fig. 4. Pl. 21.

Rule.

Rule.

Draw the diagonals A C, B C dividing the solid into triangular prisms. Then find the solidity of each of those prisms, by one of the preceding problems, and their sum will be the solidity required.

Prob. 16.

To find the convex surface of any part of a cylinder, made by a perpendicular section, fig. 5. Pl. 21.

Rule.

Multiply the length of the arc A B C by the height A D, and the product will be the curve surface.

Example.

What will be the convex surface of the section K, the length of the arc A B C being 18 inches, and the length A D 40 inches?

$18 \times 40 = 720$ square inches $=$ the curve surface required.

Prob. 17.

To find the solidity of any part of a cylinder, made by a perpendicular section, fig. 5. Pl. 21.

Rule.

Rule.

Multiply the area of the base A B C A by the height A D, and the product will be the solidity.

Example.

What will be the solidity of the part K of the cylinder E F, whose linear length of the arc A B C is 10·47 inches, the chord A C 8·6 inches, the radius A G 5 inches, and the height 30 inches?

Find the area of the base A B C A by problem 16 and 17, sect. 4. which will be 15·210 square inches. Then 15·210 × 30 = 456·3 cubic inches = the solidity required.

Prob. 18.

To find the solidities of the two parts A B, C D *of a cylinder* A B, *cut by two planes* C F, G F *forming an angle at the axis* E F, fig. 6. Pl. 21.

Rule.

From the content of the cylinder A D, subtract that of the part C F D C, and the difference will be the solidity of the part A C F B G.

Example.

What will be the content of the part C F D G C, the radius E G being 7 inches, the length G D 25 inches, and the angle C E G 95 degrees?

1. Find

1. Find the area of the base A H, of the cylinder A B, by prob 13, sect. 4. which will be 153·9384 square inches.

2. Find the area of the base C E G, of the part C F D C, by prob. 12 and 16, sect. 4, which will be 40·6226 square inches.

Then $153 \cdot 9384 \times 25 = 3848 \cdot 46 =$ content of the cylinder A B.

$40 \cdot 6226 \times 25 = 1015 \cdot 565 =$ content of the part C F D C.

$3848 \cdot 46 - 1015 \cdot 565 = 2832 \cdot 895 =$ content of the part A C F B G.

Prob. 19.

To find the convex surface of a frustum of a cylinder, fig. 7. Pl. 21.

Rule.

Multiply the circumference A B C D A, by half the sum of the least and greatest lengths A F, C E, and the product will be the surface required.

Example.

What will be the convex surface of the frustum A E, whose diameter A C is 18 inches, the length A F 10 inches, and the length C E 15 inches?

$3 \cdot 1416 \times 18 = 56 \cdot 5488 =$ circumference A B C D.

56·5488

$56{\cdot}5488 \times \frac{10 + 15}{2} = 706{\cdot}86$ square inches = surface required.

Prob. 20.

To find the solidity of the frustum of a cylinder, fig. 7. Pl. 21.

Rule.

Multiply the area of the base A B C D A, by half the greatest and the least lengths C E, A F, and the product will be the solidity.

Example.

What will be the solidity of the frustum A E, whose diameter A C is 24 inches, the length C E 36 inches, and the length A F 20 inches?

Find the area of the circle by prob. 13. sect. 4.

$24 \times 24 = 576$ = square of the diameter A C.

$\cdot 7854 \times 576 = 452{\cdot}3904$ = area of the base A B C D A.

$452{\cdot}3904 \times \frac{36 + 20}{2} = 12666{\cdot}9312$ cubic inches = solidity required.

Prob. 21.

To find the content of the solid part of the frustum of a hollow cylinder, fig. 8. Pl. 21.

Rule

Rule 1.

From the content of the frustum A B of the cylinder, subtract the content of that G D, and the difference will be the solidity (see the preceding prob.)

Rule 2.

Multiply the area of the ring A H O L, by half the sum of the greatest and the least lengths C B, A F, and the product will be the solidity (see prob. 19. sect. 4.)

Example by Rule 2.

What will be the content of the solid part of the frustum A B, whose diameter A C is 15 inches, the diameter G O 10 inches, the length C B 20 inches, and the length A F 17 inches?

$3{\cdot}1416 \times 15 = 47{\cdot}1240 =$ circumference of diameter A C.

$3{\cdot}1416 \times 10 = 31.4140 =$ circumference of diameter G O.

$\frac{15 - 10}{2} = 2.5 =$ half difference of the diameters.

$\frac{47{\cdot}1240 + 31.4160}{2} = 39.27 =$ half sum of the circumferences.

$39.27 \times 2.5 = 98.175$ square inches $=$ area of the ring A H O L.

98.175

$98.175 \times \frac{20 + 17}{2} = 1816.2375$ cubic inches = the solidity required.

Note. If it was required to find the weight of metal the frustum is made of, as for instance of cast iron: Multiply the content in solid inches, by 4.2968*, and the product will be the weight required.

Prob. 22.

To find the solidity and the weight of metal of the trunnion of a 24 *pounder, heavy gun,* fig. 9. Pl. 21.

Rule.

From the solidity of the frustum of the cylinder D F, take the content of the section C A D K C, cut off by the convexity of the second reinforce, and the difference will be the solidity.

To find the solidity of the section C A D K C, multiply the greatest thickness A B by B K, and this product again by $\frac{4}{7}$ of D C, and this last product will be the content nearly.

Example.

What will be the solidity of the trunnion of a 24 pounder heavy gun, whose diameter F G is 5.824 inches, its greatest length D G 8.64, its least length C F 6 inches; the diameter C D 6.4,

* A cubic inch of cast iron weighs 4.2968 ounces.

and

and the greatest thickness A B of the section .68 hundreths of an inch?

For the frustum D F (see prob. 20.)

$5.824 \times 5.824 = 33.918976$ = square of the diameter F G.

$33.918976 \times \frac{8.64 + 6}{2} = 248.2869$ cubic inches = solidity D F.

For the section C A D K C.

$5.824 \times .68 = 3.96032$.

$3.96032 \times \frac{6.4 \times 4}{7} = 14.5$ cubic inches = the solidity of the section C A D K C.

Then $248.2869 - 14.5 = 233.8035$ cubic inches = solidity required.

The content 233.8035 being multiplied by 5.0833*, the product will be the weight in ounces when brass, and when iron by 4.2968.

Prob. 23.

To find the solidity of a hoof, or ungula A D E C A, *of a cylinder*, fig. 10. Pl. 21.

Rule.

Multiply the surface of the right angled triangle L E C by $\frac{2}{3}$ of the diameter A D, or B E, and the product will be the solidity.

* A cubic inch of gun metal weighs 5.0833 ounces, and a cubic inch of cast iron weighs 4.2968 ounces.

Example.

Example.

It is required to find the solidity of one of the ungulas A E D C of the round turret, erected upon the middle of a batardeau, whose diameter A D is 9 feet L E 4.5 feet, and C E 4.5 feet?

$\frac{4.5 \times 4.5}{2} = 10.125$ square feet = surface of the triangle L E C.

$10.125 \times \frac{9 \times 2}{3} = 60.75$ cubic feet = the solidity required.

Prob. 24.

To find the surface of a regular pyramid, fig. 11. Pl. 21.

Rule.

Multiply the perimeter A B C D E A of the base by the length K S, and half the product will be the surface.

Example.

What will be the surface of the pentagonal pyramid S, one of its sides A B at the base being 4 feet, and the length K S 25 feet?

$4 \times 5 = 20$ = the perimeter of the base.

$\frac{20 \times 25}{2} = 250$ square feet = the surface required.

Note.

Note. The same rule is used for an irregular pyramid, fig. 12, by finding first the area of the base, and the surface of each side ASB, ASC, &c. by prob. 2 and 8. sect. 4. and their sum will be the surface required.

Prob. 25.

To find the solidity of a regular pyramid ABCES, fig. 11. Pl. 21.

Rule.

Multiply the area of the base ABCDEA, by $\frac{1}{3}$ of the perpendicular height OS, and the product will be the solidity.

Example.

What will be the solidity of the pentagonal pyramid ACES, whose linear side AB of the base is 6 feet, the perpendicular KO 4.12 feet, and the perpendicular height OS 30 feet?

$6 \times 5 = 30 =$ the perimeter.

$\frac{30 \times 4.12}{2} = 61.80$ square feet $=$ area of the base.

$\frac{61.80 \times 30}{3} = 618$ cubic feet $=$ the solidity required.

Note. In the same manner may be obtained the solidity of an irregular pyramid, fig. 12. Pl. 21.

Pl. 21. by finding the base ABDEC, according to prob. 8. sect. 4.

Prob. 26.

To find the convex surface of a right cone, fig. 13. Pl. 21.

Rule.

Multiply the circumference ADBEA of the base by the length AC, and half the product will be the convex surface.

Example.

What will be the convex surface of the cone ABC, whose circumference ADBEA is 50 inches, and the length AC 32 inches?

$\frac{50 \times 32}{2} = 800$ square inches $=$ surface required.

Prob. 27.

To find the solidity of a right cone ABC, fig. 13. Pl. 21

Rule.

Multiply the area of the base ADBEA by the perpendicular height FC, and $\frac{1}{3}$ of the product will be the solidity.

Example.

Example.

Suppose the diameter A B to be 16 inches, and the perpendicular height F C 30 inches, what will be the solidity of the cone?

$16 \times 16 = 256 =$ square of the diameter A B.

$.7854 \times 256 = 201.0624 =$ area of the base.

$\frac{201.0624 \times 30}{3} = 2010.624$ cubic inches $=$ the solidity required.

Prob. 28.

To find the solidity of an oblique pyramid, or of a cone, fig. 14, and 15. Pl. 21.

Rule.

Multiply the area of the base A B C D A by the perpendicular height E F, and $\frac{1}{3}$ of the product will be the solidity.

Example.

What will be the solidity of the square pyramid A C F A, the side A B of its base being 3 feet, and the height E F 12 feet? fig. 14.

$3 \times 3 = 9 =$ area of the base A C.

$\frac{9 \times 12}{3} = 36$ cubic feet $=$ the solidity required.

Prob. 29.

To find the surface of the frustum of a right pyramid A I, fig. 1. Pl. 22.

Rule.

Multiply the sum of the perimeters A B C D E, F H I K G, by the length L S, and half the product will be the surface.

Example.

What will be the surface of the frustum A I, of a pentagonal pyramid, whose side A B at the base is 15 inches, F H 10 inches, and the length L S 25 inches?

$15 \times 5 = 75 =$ perimeter of the base A C E.

$10 \times 5 = 50 =$ perimeter of the end F I G.

$\frac{\overline{75 + 50} \times 25}{2} = 1512.5$ square inches = the surface required.

Prob. 30.

To find the solidity of the frustum of a pyramid, fig. 1. Pl. 22.

Rule.

To the areas of the two ends, add the mean proportional* between them; then multiply their sum

* To find a mean proportional between the two ends M and N, take the square root of the product of the areas of the two ends, which will give the mean proportional plane. Or by the following proportion; as one of the sides A B of the base, is to the homologous side F H of the other end, so is the area of the base M to the mean proportional required.

sum by the perpendicular height M N, and $\frac{1}{3}$ of the product will be the solidity.

Example.

What will be the solidity of the frustum A I, of a pentagonal pyramid, whose linear side A B of its base is 6 feet, the perpendicular M S 4.12 feet, the side F H 4 feet, the perpendicular L N 2.75 feet, and the height M N 12 feet?

$6 \times 5 = 30 =$ the perimeter of the base M.

$\frac{30 \times 4.12}{2} = 61.80 =$ area of the base M.

$4 \times 5 = 20 =$ perimeter of the end N.

$\frac{20 \times 2.75}{2} = 27.50 =$ area of the end N.

$6 : 4 :: 61.80 :$ the mean proportional plane.

$\frac{61.80 \times 4}{6} = 41.20 =$ the mean proportional plane.

$61.80 + 27.50 + 41.20 = 130.50 =$ sum of the three planes.

Then $\frac{130.50 \times 12}{3} = 522$ cubic feet $=$ the solidity required.

Prob. 31.

To find the convex surface of the frustum of a right cone, fig. 2. Pl. 22.

Rule.

Multiply the sum of the perimeters of the two ends, by the length A E, and half the product will be the convex surface.

Example.

What will be the convex surface of the frustum A F, whose diameters A C, E F are 15 inches, and 9 inches, and the length A E 12 inches?

$3.1416 \times 15 = 47.1240 =$ perimeter A B C D A.

$3.1416 \times 9 = 28.2744 =$ perimeter E G F H E.

$\frac{\overline{47.1240 + 28.2744} \times 12}{2} = 452.3904$ square inches $=$ the convex surface required.

Prob. 32.

To find the solidity of the frustum of a right cone, fig. 2. Pl. 22.

Rule.

To the areas of the two ends, add the mean proportional between them*; then multiply their sum

* A mean proportional between the areas of the two ends of a frustum of a right cone, is found in the same manner as has been shewn in the note to problem 30. But for a frustum of a cone, it may more readily be obtained, by multiplying the circumference of one of the ends, by half the radius of the other end, and the product will be the mean proportional required.

sum by the perpendicular height SK, and $\frac{1}{3}$ of the product will be the solidity.

Example.

What will be the solidity of the frustum AF of a right cone, whose diameters AC and EF are 18 inches and 12 inches, and the perpendicular height SK 15 inches?

$18 \times 18 = 324 =$ square of AC.

$.7854 \times 324 = 254.4696 =$ area ABCDA.

$12 \times 12 = 144 =$ square of EF.

$.7854 \times 144 = 113.0976 =$ area EGFHE.

$3.1416 \times 18 = 56.5488 =$ circumference, or perimeter ABCDA.

$56.5488 \times 3 = 169.6464 =$ area of the mean proportional.

$254.4696 + 113.0976 + 169.6464 = 537.2136$ $=$ sum of the three planes.

Then $\frac{537.2136 \times 15}{3} = 2686.0680$ cubic inches $=$ the solidity required.

Prob. 33.

To find the content of the solid part of the frustum of a right cone, from which a cylinder has been taken, having the same axis, fig. 3. Pl. 22.

Rule 1.

From the content of the frustum of the cone, take that of the cylinder, and the difference will be the solidily (see prob. 9 and 32. sect. 5.)

Rule 2.

To the areas of the rings of the two ends, add the mean proportional between them; then multiply their sum by the perpendicular height L M, and $\frac{1}{3}$ of the product will be the solidity (see prob 19. sect. 4. and the note to prob. 32. sect. 5.)

Example.

What will be the content of the solid part of the frustum A D, from which a cylinder L I is taken, the diameters A B, C D being 12 inches and 9 inches, the diameter M I 4 inches, and the perpendicular height L M 18 inches?

$12 \times 12 = 144$ square of A B.

$.7854 \times 144 = 113.0976 =$ area A F B E A.

$9 \times 9 = 81 =$ square of C D.

$.7854 \times 81 = 63.6174 =$ area C G D H C.

$3.1416 \times 9 = 28.2744 =$ circumference C G D H C.

$28.2744 \times 3 = 84.8232 =$ area of the mean proportional.

$113.0976 + 63.6174 + 84.8232 = 261.5382$ = the sum of the three planes.

$\frac{261.5382 \times 18}{3} = 1569.2292 =$ the content of the frustum A D; see the preceding prob.

$4 \times 4 = 16 =$ square of M I.

.7854

$.7854 \times 16 = 12.5664 =$ area of one of the ends of the cylinder L I.

$12.5664 \times 18 = 226.1952 =$ content of the cylinder L I.

Then $1569.2292 - 226.1952 = 1343.034$ cubic inches $=$ the solidity required.

Prob. 34.

To find the weight of metal of the second reinforce, of a brass 24 pounder heavy gun, fig. 3. Pl. 22.

Rule.

Multiply the content in inches by 5.0833 (the weight in ounces of a cubic inch of gun metal) and the product will be the weight required.

Example.

Suppose the thickness of metal A L at the beginning of the reinforce to be 5.1 inches, the thickness C M at its extremity 4.7 inches, the diameter M I of the bore 5.824 inches, and its length L M, 22.1 inches, What will be its weight?

Find the content of the hollow frustum A D by the preceding problem, which will be 3382.59 cubic inches.

Then $3382.59 \times 5.0833 = 17194.7197$ ounces, or 1074 pounds 10.7102 ounces $=$ the weight required.

Prob.

Prob. 35.

To find the solidities of the parts A B G S, A E G S *of the frustum of a right cone* E D, *cut by the two planes* A K, G S *forming an angle at the axis* S K, fig. 4. Pl. 22.

Rule.

Find the solidity of the part A B G S, considered as the frustum of a pyramid, by prob. 30. sect. 5. and prob. 16. sect. 4. then that of the frustum of a cone E D, by prob. 32. sect. 5. Take the first content from the second, and the difference will be the solidity of the part A E G S.

Example.

What will be the solidity of each part of the frustum E D, whose diameters E B, F D are 20 inches and 12 inches, the perpendicular height S K 16 inches, and the angle A S H, or C K G 120 degrees?

$20 \times 20 = 400 =$ square of E B.

$.7854 \times 400 = 314.1600 =$ area of the base E A B H E.

$12 \times 12 = 144 =$ square of F D.

$.7854 \times 144 = 113.0976 =$ area of the end F C D G F.

$3.1416 \times 20 = 62.8320 =$ circumference A B H E A.

62.8320

$62.8320 \times 3 = 188.4960 =$ mean proportional (see the note to prob. 32. sect. 5.)

$314.1600 + 113.0976 + 188.4960 = 615.7536$ = sum of the three planes.

$\frac{615.7536 \times 16}{3} = 3284.0192 =$ solidity of the frustum E D.

$20.944 =$ arc A B H (see problem 12. sect. 4.)

$104.72 =$ area of the sector A B H S A (see prob. 16. sect. 4.)

$12.5664 =$ arc C D G (see problem 12. sect. 4.)

$37.6992 =$ area of the sector C D G K C.

$62.832 =$ mean proportional between A B H S A and C D G K C.

$104.72 + 37.6992 + 62.832 = 205.2512 =$ sum of the three planes.

$\frac{205.2512 \times 16}{3} = 1094.6731 =$ solidity of the part A B G S.

$3284.0192 - 1094.6731 = 2189.3461 =$ solidity of the part A E G S.

Prob. 36.

To find the solidity of the part ABDKEIF, *cut by the sections* D B, E F, *out of a hollow frustum of a cone* O L, *from which a cylinder* P H, *having the same axis, has been taken,* fig. 5. Pl. 22.

Rule.

Rule.

Find the content of the part A T S E K A of the frustum A L, by the preceding problem; then that of the part B T S C I F, of the cylinder B H, by prob. 18. sect 5. subtract the second content from the first, and the difference will be the solidity required.

Example.

What will be the solidity of the part B K E F, the radius T A being 29 inches, the radius S D 24 inches, the perpendicular height T S 30 inches, the radius T B or S C of the cylinder P H 14 inches, and the angle A T N or D S E 124 degrees 42 minutes?

22994.2899 = solidity of the part A T S E K A.
— 6398.7052 = solidity of the part B T S C I F.

16595.5847 = the solidity required

Note 1. The same content may also be obtained by the following rule. To the areas of the two ends B A O N F, C D K I (see prob. 19. sect. 4.) add the mean proportional between them; then multiply their sum by the perpendicular height B C, and $\frac{1}{3}$ of the product will be the solidity required.

Note 2. By one of these rules, the solidity of the revetement of the orillon of a bastion may be obtained;

obtained; as likewise the projecting part of the round towers in ancient fortification.

Prob. 37.

To find the solidity of the part LMQSKIHGL *cut by the planes* HL, IS, *out of a hollow cylinder* AB, *from which the frustum of a right cone* ED, *having the same axis, has been taken,* fig. 6. Pl. 22.

Rule.

Find the content of the part GKRPLS of the cylinder AB, by prob. 17. sect. 5. then that of the frustum of a cone HIRPMQ, by prob. 35. sect. 5. subtract the second content from the first, and the difference will be the solidity required.

Example.

What will be the solidity of the part LMQSKIHGL, the radius PL or RG being 20 inches, the radius PM 10 inches, the radius RH 15 inches, the perpendicular height PR 40 inches, and the angle LPS, or GRK 80 degrees?

11170.132 = solidity of the part GKRPLS.
— 4421.510 = solidity of the part HIRPMQ.

6748.622 = solidity required.

Note 1. The same content may also be obtained by the following rule. To the areas of the

the two ends L M Q S L, G H I K G (see the note to prob. 19. sect. 4.) add the mean proportional between them; then multiply their sum by the perpendicular height B C, and $\frac{1}{3}$ of the product will be the solidity required.

Note 2. By one of these rules the solidity of the revetement, of the concave flank of a bastion, as likewise that of the circular part of the ditch, opposite the saliant angles, of the several works of a fortification may be obtained.

Prob. 38.

To find the convex surface of a sphere, fig. 9. Pl. 19.

Rule 1.

Multiply the circumference of the sphere by the diameter, and the product will be the convex surface.

Rule 2.

Multiply the area of one of the great circles of the sphere by 4, and the product will be the convex surface.

Example.

What will be the convex surface of the sphere A L B D, its diameter A B being 20 inches.

$3.1416 \times 20 = 62.8320 =$ circumference.

62.8320

$62.8320 \times 20 = 1256.64$ square inches = surface required.

Prob. 39.

To find the diameter of a sphere, whose convex surface is given, fig. 9. Pl. 19.

Rule.

Divide $\frac{1}{4}$ of the surface of the sphere by .7854, and the square root of the quotient will be the diameter.

Example.

What will be the diameter of the sphere ALBD, whose surface is 1256.64 inches?

$\frac{1256.64}{4} = 314.16 = \frac{1}{4}$ of the surface.

$\frac{314.16}{.7854} = 400$ of which the square root is 20 = the diameter required.

Prob. 40.

To find the solidity of a sphere.

Either of the four following rules may be used to find the solidity of a sphere.

Rule 1.

Multiply the surface of the sphere by $\frac{1}{3}$ of its radius, and the product will be the solidity.

Rule

Rule 2.

Multiply the surface of the sphere by its diameter, and $\frac{1}{6}$ of the product will be the solidity.

Rule 3.

Multiply four times the area of one of the great circles of the sphere by $\frac{1}{3}$ of the radius, and the product will be the solidity.

Rule 4.

Multiply the cube of the diameter by .5236*, and the product will be the solidity.

Example 1, *by Rule* 1.

What will be the solidity of the sphere A L B D, whose diameter A B is 18 inches? fig. 9. Pl. 19.

$3.1416 \times 18 = 56.5488 =$ circumference.

$56.5488 \times 18 = 1017.8784 =$ surface of the sphere (see prob. 38. sect. 5.)

$1017.8784 \times 3 = 3053.6352$ cubic inches = the solidity required.

Example 2, *by Rule* 4.

What will be the solidity of a sphere A L B D, whose diameter A B is 20 inches? fig. 9. Pl. 19.

$20 \times 20 \times 20 = 8000 =$ cube of the diameter A B.

* See Mr. Bonnycastle's Mensuration.

$8000 \times .5236 = 4188.8$ cubic inches = solidity required.

Note. By the same rule the solidity of a hemisphere A D B A, fig. 9, may be obtained, by finding first the solidity of the sphere, and half the content will be that of the hemisphere.

Prob. 41.

To find the convex surface of the segment of a sphere A D B A, fig. 7. Pl. 22.

Rule.

Multiply the circumference of one of the great circles of the sphere, by the perpendicular C D of the segment, and the product will be the convex surface.

Example.

What will be the convex surface of the segment A D B A, the diameter F G of the sphere being 20 inches, and the perpendicular height C D 4 inches?

$3\cdot1416 \times 20 = 62\cdot8320$ = circumference of one of the great circles.

$62\cdot8320 \times 4 = 251\cdot328$ square inches = the surface required.

Note. Should the diameter of the segment be given, instead of its perpendicular height C D, find E C by prob. 4. sect. 4, which being de-

 ducted

ducted from the radius E D or E G, will give the perpendicular height C D.

Prob. 42.

To find the solidity of the sector A D B E A *of a sphere*, fig. 10. Pl. 19.

Rule.

Multiply the convex surface A D B A (see the preceding prob.) by $\frac{1}{3}$ of the radius E D, and the product will be the solidity.

Example.

What will be the solidity of the sector A D B E A, the side E B, or the radius E D being 12 inches, and the perpendicular height C D of the segment 3 inches?

$3{\cdot}1416 \times 24 = 75{\cdot}3984$ = circumference of the sphere.

$75{\cdot}3984 \times 3 = 226{\cdot}1952$ square inches = convex surface of the segment.

$226{\cdot}1952 \times 4 = 904{\cdot}7808$ cubic inches = the solidity required.

Note. Should only the diameter A B and the radius E B be given, find the height C D as has been shewn in the note to the preceding problem.

Prob. 43.

To find the solidity of the segment A D B A *of a sphere*, fig. 10. Pl. 19.

Rule

Rule 1.

From the solidity of the sector AEBDA (see the preceding prob.) take that of the cone ABE (see prob. 27. sect. 5.) and the difference will be the solidity.

Rule 2.

Multiply the area of a circle whose radius is equal to the height CD of the segment, by the radius ED of the sphere minus $\frac{1}{3}$ of the height CD, and the product will be the solidity.

Rule 3.

To three times the radius CB of its base, add the square of its height CD; multiplying the sum by the height CD, and the product by ·5236, and it will give the solidity. (See Mr. Bonnycastle's, or Dr. Hutton's Mensuration.)

Example, by Rule 2.

What will be the solidity of the segment ADBA of a sphere, whose height CD is 3 inches, and the radius ED of the sphere 9 inches?

$6 \times 6 = 36 =$ square of the diameter of a circle, whose radius CD is 3 inches.

$\cdot 7854 \times 36 = 28 \cdot 2744 =$ area of the circle.

$28 \cdot 2744 \times 9 - 1 = 226 \cdot 1952$ cubic inches $=$ solidity required.

Prob. 44.

To find the curve surface of the zone of a sphere ADHFA, fig. 11. Pl. 19.

Rule.

Multiply the circumference of one of the great circles of the sphere, by the perpendicular height KL of the zone ADHFA, and the product will be the curve surface.

Example.

What will be the curve surface of the zone AH, the diameter of the sphere being 30 inches, and the perpendicular height KL 14 inches?

$3{\cdot}1416 \times 30 = 94{\cdot}2480 =$ circumference of one of the great circles of the sphere.

$94{\cdot}2480 \times 14 = 1319{\cdot}4720$ square inches $=$ surface required.

Prob. 45.

To find the solidity of the frustum, or zone of a sphere ADHFA, fig. 11. Pl. 19.

Rule 1.

From the content of the sphere AMHNA, subtract that of the sum of the two segments ADMA, FHNF, and the difference will be the solidity.

Rule 2.

To the sum of the squares of the two radii FI, AK of the two ends, add $\frac{1}{3}$ of the square of their distance

distance KL; multiply this sum by the said distance KL, and the product again by 1·5708*, and it will give the solidity required.

Example, by Rule 2.

What will be the solidity of the zone ADHFA, the radius AK being 10 inches, the radius FL 8 inches, and the height or distance KL 9 inches?

$10 \times 10 = 100 =$ square of AK.
$8 \times 8 = 64 =$ square of FL.
$9 \times 9 = 81 =$ square of KL.
$\frac{81}{3} = 27 = \frac{1}{3}$ of the square of FG.

$\overline{100 + 64 + 27} \times 9 \times 1{\cdot}5708 = 2700{\cdot}2052$ cubic inches = the solidity required.

Prob. 46.

The solidity of a sphere being given, to find its diameter, fig. 9. Pl. 19.

Rule.

Divide the solidity of the sphere by ·5236, and the cube root of the quotient will be the diameter.

Example.

The solidity of the sphere ALBDA being 113·0976 solid inches, what will be its diameter AB?

* See Mr. Bonnycastle's Mensuration.

$\frac{113.0976}{.5236} = 216$ of which the cube root is 6 inches = the diameter required.

Prob. 47.

To find the weight of an iron shot, its diameter being given.

Rule.

Take $\frac{1}{8}$ of the cube of the diameter, and $\frac{1}{8}$ of that eighth, and the sum of these two quotients will be the weight required in pounds*.

Example.

What is the weight of an iron shot, whose diameter is 3·5 inches?

$3{\cdot}5 \times 3{\cdot}5 \times 3{\cdot}5 = 42{\cdot}875 =$ cube of the diameter.

$\frac{42.875}{8} = 5{\cdot}359 =$ 1st quotient.

$\frac{5{\cdot}359}{8} = {\cdot}669 = 6{\cdot}028 = 6$ pounds nearly.

Question.

What is the weight of an iron shot, whose diameter is 6·7 inches? Answer 42 pounds.

Prob. 48.

To find the weight of a leaden ball, its diameter being given.

* See Mr. Bonnycastle's Mensuration.

Rule.

Rule.

Take $\frac{1}{3}$ of the cube of the diameter, and from it subtract $\frac{1}{3}$ of this third, and the remainder will be the weight required nearly.

Example.

What is the weight of a leaden ball, whose diameter is 3·3 inches?

$3{\cdot}3 \times 3{\cdot}3 \times 3{\cdot}3 = 35{\cdot}937$ cube of the diameter.

$\frac{35{\cdot}937}{3} = 11{\cdot}979 =$ 1st third.

$\frac{11{\cdot}979}{3} = 3{\cdot}993 =$ 2d third.

$11{\cdot}979 - 3{\cdot}993 = 7{\cdot}986 = 8$ pounds nearly.

Prob. 49.

To find the diameter of an iron shot, its weight being given.

Rule.

Multiply the weight by 7, and to the product add $\frac{1}{9}$ of the weight, and the cube root of the sum will be the diameter in inches.

Example.

What is the diameter of an iron shot, whose weight is 18 pounds?

$18 \times 7 = 126.$

$\frac{18}{9} = 2.$

$126 + 2 = 128$ of which the cube root is $5{\cdot}039 = 5{\cdot}04$ inches nearly.

Prob. 50.

To find the diameter of a leaden ball, its weight being given.

Rule.

To 4 times the weight, add half the weight, and $\frac{3}{100}$ of half the weight, and the cube root of this sum will be the diameter in inches nearly.

Example.

What is the diameter of a leaden ball, whose weight is 8 pounds?

$8 \times 4 = 32.$

$\frac{8}{2} = 4 = \frac{1}{2}$ of 8.

$\frac{4}{100} \times 3 = {\cdot}12 = \frac{3}{100}$ of 4.

$32 + 4 + {\cdot}12 = 36{\cdot}12$ of which the cube root is $3{\cdot}3$ inches nearly.

Prob. 51.

To find the weight of an iron shell, its interior and exterior diameter being given.

Rule.

Rule.

Take $\frac{1}{8}$ of the difference of the cube of the diameters in inches, and $\frac{1}{8}$ of that eighth, and their sum will be the weight in pounds.

Example.

What is the weight of a shell, whose exterior diameter is 12·85 inches, and interior diameter 8·75 inches?

Note. A shell of those dimensions is called a 13 *inch shell.*

$12{\cdot}85 \times 12{\cdot}85 \times 12{\cdot}85 = 2121{\cdot}824125 =$ cube of the exterior diameter.

$8{\cdot}75 \times 8{\cdot}75 \times 8{\cdot}75 = 669{\cdot}921875 =$ cube of the interior diameter.

$2121{\cdot}824125 - 669{\cdot}921875 = 1451{\cdot}902250$ = difference of the cubes of the diameters.

$\frac{1451.902250}{8} = 181{\cdot}487781 =$ 1st eighth

$\frac{181.487781}{8} = 22.685972 =$ 2d eighth.

$181.487781 + 22.685972 = 204.173753 =$ 204.17 pounds nearly.

Prob. 52.

To find the quantity of powder a shell will contain.

Rule.

Rule.

Divide the cube of the interior diameter in inches by 57.6*, and the quotient will be the weight in pounds nearly.

Example.

What quantity of powder is required to fill a 13 inch shell, whose interior diameter is 8.75 inches?

$8.75 \times 8.75 \times 8.75 = 669.921875 =$ cube of the interior diameter.

$\frac{669.921875}{57.6} = 11.6$ pounds nearly.

Prob. 53.

To find the side of a cubical box, to contain a given quantity of gunpowder.

Rule.

Divide the given quantity of powder by 55, and the cube root of the quotient will be the side of the box, in feet.

Example.

What will be the side of a cubical box to hold 400 pounds of powder?

* 57.6 is the number of pounds of gunpowder contained in a cubic foot, when shaken, and 55 pounds only when not shaken.

$\frac{400}{55} = 7.272727 =$ quotient, of which the cube root is 1.93 feet, or 23.16 inches nearly.

Prob. 54.

The height of a square box being given, what will be the length of the side to hold a given quantity of gunpowder.

Rule.

Divide the given quantity of powder by the product of 55 multiplied by the given height, and the square root of the quotient will be the side of the box.

Example.

The given height of a box is 1 foot, what will be the length of the side, to contain 380 pounds of gunpowder?

$55 \times 1 = 55.$

$\frac{380}{55} = 6.9$ of which the square root is 2.6267 feet, or 31.5 inches for the length of the required side.

Question.

What will be the length of the side of a box, its height being 9 inches, or .75 of a foot, to contain 300 pounds of gunpowder? Answer 2.696 feet = 32.3 inches nearly.

Prob.

Prob. 55.

To find the quantity of powder to fill the chamber of a mortar, or of a howitzer.

Rule.

Multiply the content of the chamber in inches by 55, and divide the product by 1728*, and the quotient will be the quantity of powder in pounds.

Note. The chamber of a mortar, or of a howitzer is formed of a hollow frustum of a right cone ABCE, and of a hollow hemisphere nearly CDEC, fig. 8. Pl. 22.

Example.

What will be the quantity of powder to fill the chamber ABEDCA of a 13 inch sea-mortar, in which the diameter AB is 9·6 inches; the diameter CE 6·8 inches, and the length DG 21·55 inches?

Find the content of the chamber by prob. 26, and by the note of prob. 33. sect. 5, which will be 1040·13844322.

Then $\frac{1040.13844322 \times 55}{1728} = 33$ pounds nearly = the quantity of powder required.

* 1728 is the number of cubic inches contained in a cubic foot.

Prob.

Prob. 56.

To find the solidity of a flat ring or hoop, fig. 9. Pl. 22.

Rule.

Multiply half the sum of the interior and exterior circumference of the hoop, by its thickness, and this product again by its breadth, and it will give the solidity required.

Example.

What will be the solidity of a ring, whose exterior diameter AB is 18 inches, the interior diameter CD 16.4 inches, and its breadth AE 1.4 inch?

$3.1416 \times 18 = 56.54488 =$ exterior circumference.

$3.1416 \times 16.4 = 51.52224 =$ interior circumference.

$\frac{18 - 16.4}{2} = .8$ inch $=$ thickness AC.

$\frac{56.54488 + 51.52224}{2} \times .8 \times 1.4 = 60.5176$ cubic inches $=$ the solidity required.

Note. If the above dimensions are supposed to be those of the base ring of a brass gun, its weight in ounces will be obtained by multiplying the content 60.5176 cubic inches by 5.0833*.

That is, $60.5176 \times 5.0833 = 307.621$ ounces, or 19 lb. 3.62 oz. nearly.

* A cubic inch of gun metal weighs 5.0833 ounces.

Prob.

Prob. 57.

To find the convex surface of a cylindric ring, fig. 10. Pl. 22.

Rule.

Multiply half the sum of the two circumferences AEFA and CGHC, by the circumference ABCDA of the thickness of the ring, and the product will be the convex surface.

Example.

What will be the surface of the ring AEF, whose interior diameter CI is 10 inches, and its thickness AC 3 inches?

$AC + IL + CI = AL = 3 + 3 + 10 = 16$ inches.

$3.1416 \times 16 = 50.2656 =$ circumference AEFA.

$3.1416 \times 10 = 31.4160 =$ circumference CGHC.

$\frac{50.2656 + 31.4160}{2} = 40.8408 =$ half the sum of the two circumferences.

$3.1416 \times 3 = 9.4248 =$ circumference ABCDA.

Then $40.8408 \times 9.4248 = 384.9637$ square inches $=$ the convex surface required.

Note 1. In the same manner the surface of any part of a ring as AG, or GLH, may be obtained (see prob. 12 and 18. sect. 4.)

Note

Note 2. If the ring is cut horizontally into halves, by the radius A O, the surface of each will be equal to half the surface of the ring.

Note 3. By the same rule may also be obtained, the concave surface of any semicircular turning arch.

Prob. 58.

To find the solidity of a cylindric ring, fig. 10. Pl. 22.

Rule.

Multiply half the sum of the two circumferences A E F A and C G H C, by the area of the circle A B C D A, and the product will be the solidity.

Example.

What will be the solidity of the ring A E F A, whose interior diameter C I is 8 inches, and thickness A C 3 inches?

$AC + CI + IL = AL = 3 + 8 + 3 = 14.$

$3.1416 \times 8 = 25.1328 =$ circumference C G H C.

$3.1416 \times 14 = 43.9824 =$ circumference A E F A.

$\frac{25.1328 + 43.9824}{2} = 34.5576 =$ half the sum of the two circumferences.

.7854

$.7854 \times 9 = 7.0686 =$ area of the circle A B C D A.

$34.5576 \times 7.0686 = 244.274$ cubic inches = solidity required.

Prob. 59.

To find the exterior convex surface of a semi-cylindric ring, fig. 10. Pl. 22.

Rule.

To the radius O P add 14 times the 33d part of the radius A P; multiply the circumference answering to that radius, by the semi-circumference B A D, and the product will be the convex surface.

Example.

What will be the convex surface of a torus, whose radius O P is 20 inches and the radius A P of the semi-circle 3.3 inches?

The $\frac{14}{33}$ of 3.3 inches = 1.4 inches = P Q,

$20 + 1.4 = 21.4$ inches = radius O Q, and twice this will 42.8 inches for its diameter.

$3.1416 \times 42.8 = 134.46048 =$ circumference.

$3.1416 \times 3.3 = 10.36728.$

$134.46048 \times 10.36728 = 1393.98944$ square inches = the surface required.

Note. The same rule is to be observed in finding the convex surface of a ring, whose sec-tion

tion APB or APD is a quadrant, by adding to the radius OP, the $\frac{14}{33}$ of the radius AP.

Prob. 60.

To find the solidity of a ring, whose section BAD *is a semi-circle towards the outside of the ring,* fig. 10. Pl. 22.

Rule.

To the radius OP add 14 times the 33d part of the radius AP; multiply the circumference answering to that radius, by the surface of the semi-circle BAD, and the product will be the solidity.

Example.

What will be the solidity of a torus, the radius OP being 20 inches, and the radius AP 3·3 inches?

The $\frac{14}{33}$ of 3·3 inches = 1·4 inches = PQ.

20 + 1.4 = 21·4 = radius OQ, and twice this will be 42·8 for the diameter.

3·1416 × 42·8 = 134·46048 = circumference.

6·6 × 6·6 = 43·56 = square of DB.

$\frac{.7854 \times 43.56}{2}$ = 17·106012 = area of the semicircle BAD.

134·46048 × 17·106012 = 2300·0826 cubic inches = the solidity required.

Note 1. The astragal of a gun being a semi-cylindric ring, its solidity will be found in the same manner; and if the weight thereof is required, its content, in cubic inches, must be multiplied by 5·0833* (if brass) and the product will be the weight in ounces.

Note 2. In the same manner the solidity of a ring may be obtained, whose section is a quadrant, as A B P or A D P.

Prob. 61.

To find the interior convex surface of a semi-cylindric ring, fig. 10. Pl. 22.

Rule.

From the radius O P, subtract $\frac{14}{33}$ of the radius C P, and with this radius find a circumference, which multiply by the semi-circumference B C D, and the product will be the convex surface.

Example.

What will be the interior convex surface of a semi-cylindric ring, whose radius O C is 25 inches, and the radius P C 6·6 inches?

$25 + 6{\cdot}6 = 31{\cdot}6 =$ O P; from this radius subtract $\frac{14}{33}$ of 6·6 inches, which is 2·8.

Then $31{\cdot}6 - 2{\cdot}8 = 28{\cdot}8 =$ radius required.

* A cubic inch of gun metal weighs 5.0833 ounces.

$28 \cdot 8 \times 2 = 57 \cdot 6 =$ diameter.

$3 \cdot 1416 \times 57 \cdot 6 = 180 \cdot 95616 =$ circumference.

$3 \cdot 1416 \times 6 \cdot 6 = 20 \cdot 73456 =$ semi-circumference B C D.

$180 \cdot 95616 \times 20 \cdot 73456 = 3752 \cdot 04636$ square inches $=$ surface.

Note. The same rule must be observed in finding the convex surface of a ring, whose section C B P or C D P is a quadrant, by subtracting from the radius O P, $\frac{14}{33}$ of the radius C P.

Prob. 62.

To find the solidity of a ring, whose section B C D, *is a semi-circle towards the centre of the ring,* fig. 10. Pl. 22.

Rule.

From the radius O P, subtract $\frac{14}{33}$ of the radius C P, and multiply the circumference answering to this radius, by the area of the semi-circle B C D, and the product will be the solidity.

Example.

What will be the solidity of the semi-cylindric ring B D C G H C, whose radius C O is 25 inches, and the radius C P 6·6 inches?

$25 + 6 \cdot 6 = 31 \cdot 6 =$ O P; from this radius subtract $\frac{14}{33}$ of 6·6 inches, which is 2·8.

Then $31.6 - 2.8 = 28.8 =$ radius required.

$28.8 \times 2 = 57.6 =$ diameter.

$3.1416 \times 57.6 = 180.95616 =$ circumference.

$3.1416 \times 6.6 = 20.73456 =$ circumference B C D.

$\frac{20.73456 \times 6.6}{2} = 68.424 =$ area of the semicircle B C D B.

$180.95616 \times 68.424 = 12381.7443$ cubic inches = the solidity required.

Note 1. In the same manner, the solidity of a ring may be obtained, whose section B P C, or D P C is a quadrant.

Note 2. The same rules are also made use of to find the surface and the solidity of any part of a ring, as A C G F, B P C G, &c. (see prob. 12. sect. 4.)

Prob. 63.

To find the surface of an ogee, fig. 11 and 12. Pl. 22.

Rule.

Multiply the length of the arcs A B, B C, by the length D E, and the product will be the surface.

Example.

What will be the surface of the ogee of a cornice, whose arcs A B, B C are quadrants, of which the

the radius B F, or A G is 3 inches, and the length E D 5 feet, or 60 inches?

A B + B C is equal to a semi-circumference, of which A G, or C F is the radius.

Then $3.1416 \times 3 = 9.4248$.

$60 \times 9.4248 = 565.488$ square inches = the surface.

Note. When the ogee is described by equilateral triangles, as fig. 13, or by isosceles triangles, as fig. 14, of which the radius A B is given, the length of the arcs B C, B D will be obtained, by prob. 12, sect. 4.

Prob. 64.

To find the solidity of an ogee D H, fig. 11, *whose perpendicular section is represented by* fig. 12. Pl. 22.

Rule.

Through the point of junction B of the two arcs A B, C B, draw K L parallel to A I; then multiply the surface of this parallelogram A K L I, by the length D E, and the product will be the solidity.

Example.

What will be the solidity of an ogee, whose length D E is 10 feet or 120 inches, its projection I C 6 inches, and its height A I, or H E 6 inches?

$\frac{IC}{2} = IL = 3$ inches.

$IL \times AI = 3 \times 6 = 18$ square inches = surface.

$DE \times 18 = 120 \times 18 = 2160$ cubic inches = the solidity.

Note. In the same manner is obtained the solidity of an ogee, described by equilateral, or isosceles triangles, as fig. 13 and 14. Pl. 22.

Prob. 65.

To find the surface of an ogee revolving about an axis A B, *whose arcs* C D, D E *are quadrants*, fig. 1. Pl. 23.

Rule.

Multiply the length of the two arcs C D, D E, by the circumference, passing through the point of junction D of the arcs C D, D E, or by the mean proportional between the circumferences of the two ends G E, H C, and the product will be the surface.

Example.

What will be the surface of an ogee, whose diameter D F is 18 inches, and the radius I C, or K E 2 inches?

$3.1416 \times 18 = 56.5488 =$ circumference.

$DC + DE$ is equal to a semi-circumference, whose radius is $IC = 2$ inches.

Then

Then $3.1416 \times 2 = 6.2832 =$ length of the two arcs C D, D E.

$56.5488 \times 6.2832 = 255.30742$ square inches = the surface required.

Note. In the same manner the surface of an ogee is obtained, when described by equilateral, or isosceles triangles.

Prob. 66.

To find the solidity of an ogee E H, fig. 1. Pl. 23.

Rule 1.

Multiply the perpendicular height A B, by the area of the circle F D, and the product will be the solidity.

Rule 2.

Multiply the perpendicular height A B, by the product of the circumference of the one end by half the radius of the other end, and this last product will be the solidity.

Rule 3.

To the areas of the two ends, add the mean proportional between them; multiply the sum by the perpendicular height A B, and $\frac{1}{3}$ of the product will be the solidity.

Example.

What will be the solidity of the ogee E H, its diameter C H being 14 inches, the diameter E G 22 inches, and the height A B 4 inches?

By Rule 2.

$3 \cdot 1416 \times 14 = 43 \cdot 9824 =$ circumference of the diameter C H.

$\frac{AE}{2} = \frac{11}{2} = 5 \cdot 5 =$ half the radius A E.

$43 \cdot 9824 \times 5 \cdot 5 = 241 \cdot 9032 =$ mean proportional.

$241 \cdot 9032 \times 4 = 967 \cdot 6128$ cubic inches $=$ the solidity.

Note 1. In the same manner may be obtained the solidity of a circular ogee, described by equilateral, or isosceles triangles.

Note 2. In the same manner may also be obtained the weight of any ogee of a piece of ordnance, by multiplying the content in cubic inches by 5·0833, when brass; and when iron by 4·2968.

Note 3. A cubic inch of gun metal weighs 5·0833 ounces, and a cubic inch of cast iron weighs 4·2968 ounces.

Prob. 67.

To find the concave surface of the solid A G H C, fig. 2. Pl. 23.

Rule.

Rule.

Subtract $\frac{14}{33}$ of the difference of the two radii EC — DA, from the radius DF; with this difference, as radius, find a circumference, which multiply by the area of the semi-segment CAFC, and the product will be the surface.

Example.

What will be the surface of the swell of the muzzle of a piece of ordnance, whose greatest diameter CH is 15·17 inches, its less diameter AG 11·6 inches, its height DE 9·25 inches, the radius AB 29 inches, and the angle ABC 19 degrees 30 minutes?

$EC - DA = AF = 7.585 - 5.8 = 1.785 =$ difference of the two radii.

$\frac{14\,AF}{33} = \frac{14 \times 1.785}{33} = 0.757 = PF.$

$DF - PF = DP = 7.585 - 0.757 = 6.828$ $=$ radius.

$6.828 \times 2 = 13.656 =$ diameter.

$3.1416 \times 13.656 = 42.90169 =$ circumference.

The arc $AC = 9.87$ inches (see problem 12. sect. 4.)

Then $42.90169 \times 9.87 = 423.43967$ square inches $=$ surface.

Prob.

Prob. 68.

To find the content of the solid AGHC, fig. 2. Pl. 23.

Rule.

1. Find the content of the cylinder FKHC, by prob. 9. sect. 5.

2. Multiply the area of the semi-segment CAFC, by the circumference of a circle, whose radius is $DF - \frac{14\,AF}{33}$; subtract the product from the content of the cylinder FKHC, and the difference will be the solidity.

Example.

What will be the solidity of the figure AGHC, whose diameter CH is 15.17 inches, the diameter AG 11.6 inches, the perpendicular height DE 9.25, the radius BA 29 inches, and the angle ABC 19 degrees 30 minutes?

For the cylinder FKHC.

$15.17 \times 15.17 = 230.1289 =$ square of CH.

$.7853 \times 230.1289 = 180.73324 =$ area of the circle.

$180.73324 \times 9.25 = 1671.78245$ cubic inches $=$ solidity of the cylinder (see prob. 7. sect. 5.)

For

For the area of the semi-segment FACF.

From the area of the sector BACB, take the surface of the right-angled triangle BFC, and the difference will be the area of the semi-segment FACF (see prob. 2 and 16, sect. 4.)

2 BA $= 29 \times 2 = 58 =$ diameter.

$3.1416 \times 58 = 182.2128 =$ circumference.

360 : 182.2128 : : 19° 30′ : arc AC $= 9.87$ inches.

$\frac{BA \times AC}{2} = \frac{29 \times 9.87}{2} = 143.115 =$ area of the sector ABCA.

BA — AF $=$ BF $= 29 - 1.785 = 27.215$.

$\frac{BF \times FC}{2} = \frac{27.215 \times 9.25}{2} = 125.87 =$ area of the triangle BFCB.

$143.115 - 125.87 = 17.245 =$ area of the semi-segment FACF.

FP $= \frac{14 FA}{33} = 0.757$.

DF — FP $=$ DP $= 7.585 - 0.757 = 6.828 =$ radius DP.

2 DP $= 6.828 \times 2 = 13.656 =$ diameter PL.

$3.1416 \times 13.656 = 42.90169 =$ circumference.

$42.90169 \times 17.245 = 739.83963$ cubic inches.

Then $1671.78245 - 739.83963 = 931.9428 =$ the solidity required.

Note.

Note. The same problem is made use of to find the content, and the weight of metal of the swell of the muzzle of a piece of ordnance; by deducting from the solid A H, the content of the cylinder L N, whose diameter is that of the calibre, and its height equal to the length of the section D E; which difference being multiplied by 5.0833, the product will be the weight in ounces, when brass, and when iron by 4.2968.

Prob. 69.

To find the content and the weight of a piece of ordnance, fig. 3. Pl. 23.

Rule.

Divide the length of the gun into as many sections as may be found necessary, by lines C D, E F, G H, I K, L M, &c. drawn perpendicular to the axis A B. Find the content of each part by problem 9, 12, 20, 22, 34, 43, 56, 60, 62, 66, and 68, sect. 5. and from their sum, subtract the content of a cylinder, whose length is equal to that of the bore, and its diameter equal to that of the calibre of the piece; multiply the difference (if it be a brass gun) by 5.0833, and the product will be the weight in ounces, which being divided by 16, will reduce it into pounds, and again by 112 will be hundred weights.

Prob.

Prob. 70.

To find the solidity of a spheroid, fig. 12. Pl. 19.

Rule.

Multiply the area of a circle, whose diameter is C D, by $\frac{2}{3}$ of the transverse A B, and the product will be the solidity.

Example.

What will be the solidity of the spheroid A C B D A, its transverse A B being 90 inches, and its conjugate C D 70 inches?

$70 \times 70 \times .7854 = 3848.4600 =$ area of the circle C D.

$\frac{90 \times 2}{3} = 60 = \frac{2}{3}$ of A B.

Then $3848.4600 \times 60 = 230907.6$ cubic inches = the solidity required.

Prob. 71.

To find the content of a cask, fig. 4. Pl. 23.

Rule.

Multiply half the sum of the areas of the two interior circles, that is the greater C D and the least E F, by the interior length A B, and the product will be the required content nearly.

Example.

Example.

What will be the content of the cask A D B C, its greatest interior diameter C D being 24 inches, its least interior diameter E F 20 inches, and the interior length A B 30 inches?

$24 \times 24 \times .7854 = 452.3904 =$ area of the circle C D.

$20 \times 20 \times .7854 = 314.1600 =$ area of the circle E F.

$\frac{452.3904 + 314.1600}{2} = 383.2752 =$ half sum.

Then $383.2752 \times 30 = 11498.2560 =$ the content; which being divided by 282* will give the number of gallons contained in the cask.

That is, $\frac{11498.2560}{282} = 40.7739$ gallons, or 40 gallons and 6 pints nearly.

Prob. 72.

To find the solidity of a paraboloid, fig. 5. Pl. 23.

Rule.

Multiply the area of the base A B C D A by the altitude E F, and half the product will be the solidity.

* A gallon of beer contains 282 cubic inches, a gallon of wine 231 cubic inches, and a gallon for grains, meals, &c. contains 272.25 cubic inches.

Example.

Example.

What will be the solidity of the paraboloid A B C F D, its diameter A C being 40 feet, and height E F 25 feet?

$40 \times 40 \times .7854 = 1256.6400 =$ area of the base.

$\frac{1256.6400 \times 25}{2} = 31416$ cubic feet $=$ the solidity required.

SECT. VI.

CONSTRUCTION AND MENSURATION OF THE FIVE REGULAR SOLIDS.

DEFINITIONS.

1. *REGULAR solids*, are those terminated by regular and equal planes, of which there are five; the *tetraedron*, *hexaedron*, or *cube*, *octaedron*, *dodecaedron* and *icosaedron*.

2. The *tetraedron*, or *regular triangular pyramid*, has four triangular faces, as fig. 6. Pl. 23.

3. The *hexaedron*, or *cube*, has six square faces, as fig. 9.

4. The *octaedron*, has eight triangular faces, as fig. 11.

5. The

5. The *dodecaedron*, has twelve pentagonal faces, as fig. 13.

6. The *icosaedron*, has twenty triangular faces, as fig. 15.

METHOD OF CONSTRUCTING THE FIVE REGULAR SOLIDS, WITH CARD PASTEBOARD.

Prob. 1.

To construct the tetraedon, fig. 6 and 7. Pl. 23.

On a piece of pasteboard, describe an equilateral triangle A B C; bisect each side in D, E, F, and draw the lines D E, D F, E F. Then these lines being cut half through, so that their faces may be turned up, and glued together, will form the tetraedron G H I K.

Prob. 2.

To construct a regular hexaedron, fig. 8 and 9. Pl. 23.

Describe a square A B C D; produce its sides, on which make the squares A I, A G, C E, C F and F K, equal to the square A C. Then proceed as in the preceding problem, and the required hexaedron L N will be formed.

Prob.

Prob. 3.

To construct a regular octaedron, fig. 10 and 11. Pl. 23.

Draw a line C B, and divide it into three equal parts. On the two thirds A B and C F describe the equilateral triangles A D B, C E F; bisect the sides A D, D B, C E, E F, and join these points of bisection by the lines F I, I H, &c. Then proceed as in problem first, and the required octaedron K M will be formed.

Prob. 4.

To construct a regular dodecaedron, fig. 12 and 13. Pl. 23.

On a given line A B describe the regular pentagon A B C D E, and on each side of it, those F, G, H, I, K, each equal and similar to the first. On L M construct the regular pentagon N, on O P the pentagon R, and in like manner S, T, Q, V. Then proceed according to problem first, and the required dodecaedron L M, fig. 13, will be formed.

Prob. 5.

To construct the regular icosaedron, fig. 14 and 15. Pl. 23.

Describe an equilateral triangle A B C; produce A B towards D, and through C draw C E parallel thereto; make A B and C E each equal to five times A B. Through the points F, G, K, &c. draw lines parallel to A C, produced both ways indefinitely; draw likewise through the points H, I, L, &c. parallels to B C, and by their intersections, twenty equilateral triangles will be obtained. Then the lines being cut half through, so as to be turned up and glued together, will form the required icosaedron D E, fig. 15.

Prob. 6.

The diameter of a sphere being given, to find the linear sides of the five regular solids, in order to be inscribed, or to be cut out from the given sphere, fig. 16. Pl. 23.

Let A B be the diameter of a given sphere; bisect it, and from the point of bisection C, as a centre, and with C A as radius, describe the semicircle A D B. Take A H equal to two thirds of A B. From C and H draw C D, H E perpendicular to A B. Join A E, B E, B D; then A E will be the linear side of the tetraedon; B E the side of the hexaedron, and B D the linear side of the octaedron. Draw A I perpendicular and equal to A B, and join C I. Through the intersection F draw A F, which will be the linear side of the icosaedron.

dron. Then cutting B E into extreme and mean proportion (by prob. 16, sect. 1.) and the part B G will be the linear side of the dodecaedron.

Prob. 7.

To find the surface of one of the five regular solids.

Rule.

Multiply the number of faces by the area of one of them, and the product will be the area; or by the following proportion, as 1 is to the square of the linear edge of the given solid,

$$\text{so is} \left\{ \begin{array}{r} 1.73205 \\ 6.00000 \\ 3.46410 \\ 20.64577 \\ 8.66025 \end{array} \right\} \begin{array}{c} \text{to the} \\ \text{surface} \\ \text{of the} \end{array} \left\{ \begin{array}{l} \text{Tetraedron.} \\ \text{Hexaedron.} \\ \text{Octaedron.} \\ \text{Dodecaedron.} \\ \text{Icosaedron.} \end{array} \right.$$

Example 1.

What will be the surface of the regular tetraedron H K, fig. 7. Pl. 23. *whose linear edge* G H *is* 6 *inches?*

$6 \times 6 = 36 =$ square of G H.

$1 : 36 :: 1.73205 :$ the area.

$1.73205 \times 36 = 62.35380$ square inches $=$ area required.

Example 2.

What will be the surface of a regular hexaedron L N, fig. 9. Pl. 23. *whose linear side* L M *is* 5 *inches?*

$5 \times 5 = 25 =$ square of L M.

1 : 25 : : 6.00000 : the area.

$6.00000 \times 25 = 150$ square inches $=$ surface required.

Example 3.

What will be the surface of a regular octaedron K M, fig. 11, *whose linear edge* L M *is* 4 *inches?*

$4 \times 4 = 16 =$ square of L N.

1 : 16 : : 3.46410 : the surface.

$3.46410 \times 16 = 55.42560$ square inches $=$ surface required.

Example 4.

What will be the surface of a regular dodecaedron L M, fig. 13, *whose linear edge* A B *is* 7.5 *inches?*

$7.5 \times 7.5 = 56.25 =$ square of A B.

1 : 56.25 : : 20.64577 : the surface.

$20.64577 \times 56.25 = 1161.3245625$ square inches $=$ the surface required.

Example 5.

What will be the surface of a regular icosaedron D E, fig. 15, *whose linear edge* C F *is* 3 *inches?*

$3 \times 3 = 9 =$ square of C F.

1 : 9 : : 8.66025 : the surface.

8.66025

$8.66025 \times 9 = 77.94225$ square inches = surface required.

Prob. 8.

To find the solidity of one of the five regular solids.

Rule.

Make the following proportion, as 1 is to the cube of the linear edge of the given solid,

$$\text{so is} \left\{ \begin{array}{l} 0.117851 \\ 1.000000 \\ 0.471404 \\ 7.663119 \\ 2.181695 \end{array} \right\} \begin{array}{c} \text{to the} \\ \text{solidity} \\ \text{of the} \end{array} \left\{ \begin{array}{l} \text{Tetraedron.} \\ \text{Hexaedron.} \\ \text{Octaedron.} \\ \text{Dodecaedron.} \\ \text{Icosaedron.} \end{array} \right.$$

Example 1.

What will be the solidity of a regular tetraedron H K, fig. 7. Pl. 23, *whose linear edge* G H *is* 5 *inches?*

$5 \times 5 \times 5 = 125 =$ cube of G H.

$1 : 125 :: 0.117851 :$ the solidity.

$0.117851 \times 125 = 14.731375$ cubic inches = solidity required.

Example 2.

What will be the solidity of a regular octaedron K M, fig. 11, *whose linear edge* L N *is* 4 *inches?*

$4 \times 4 \times 4 = 64 =$ cube of L N.

1 : 64

1 : 64 : : 0.471404 : the solidity.

0.471404 × 64 = 30.169856 cubic inches = solidity required.

Example 3.

What will be the solidity of a regular dodecaedron L M, fig. 13, *whose linear edge* A B *is* 6 *inches?*

6 × 6 × 6 = 216 = cube of A B.

1 : 216 : : 7.663119 : the solidity.

7.663119 × 216 = 1655.233704 cubic inches solidity required.

Example 4.

What will be the solidity of a regular icosaedron D E, fig. 15, *whose linear edge* C F *is* 3 *inches?*

3 × 3 × 3 = 27 = cube of C F.

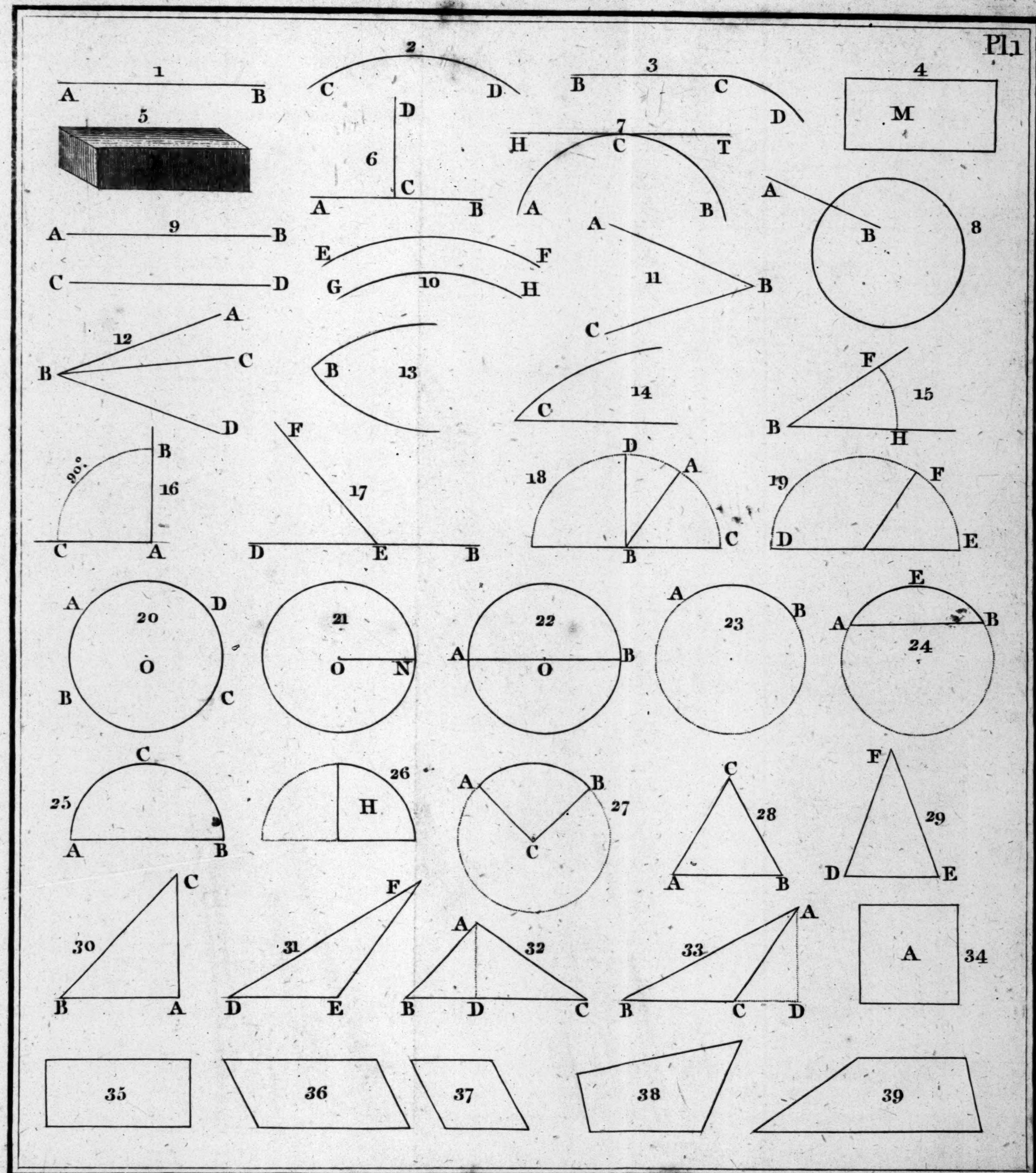
1 : 27 : : 2.181695 : the solidity.

2.181695 × 27 = 58.905765 cubic inches = = solidity required.

FINIS.

ERRATA.

Page 9. line 13; for, fig. 22, read fig. 2.
52. line 11; for, D G, read B G.
86. line 9; for, 400 square, read 400 = square.
id. line 10; for, 289 square, read 289 = square.
116. line 9; for, 400 — 314.1600, read 400 = 314.1600.
125. line 18; for, 31.4140, read 31.4160.
~~146. last line; for, Pl. 9, read Pl. 19.~~
151. line 8; for, 35.937 cube, read 35.937 = cube. 9
160. line 20; for, will 42.8, read will be 42.8.
170. line 4 from bottom; for, .7853, read .7854.
175. line 5 from bottom; for, fig. 6, read fig. 7.



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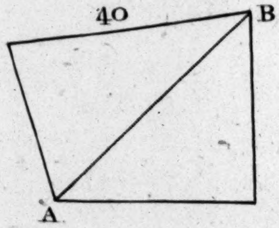

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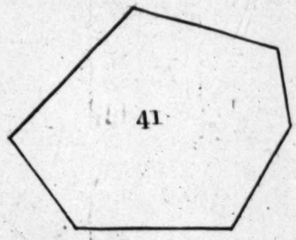

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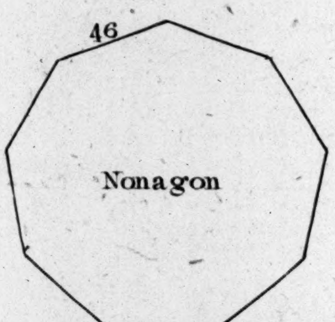

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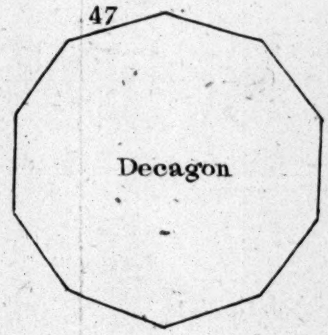

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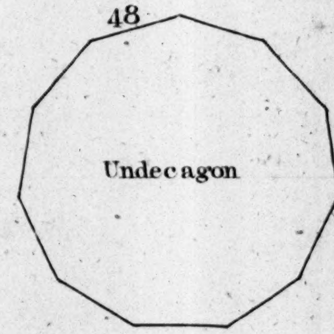

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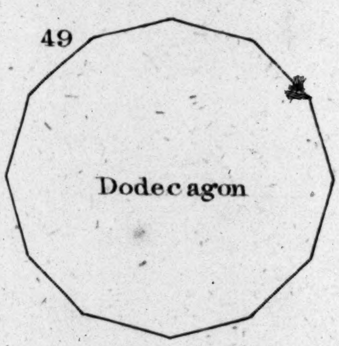

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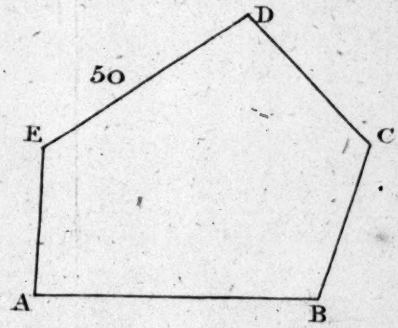

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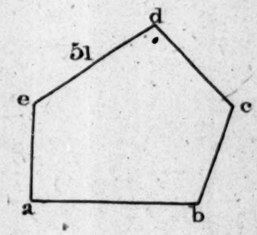

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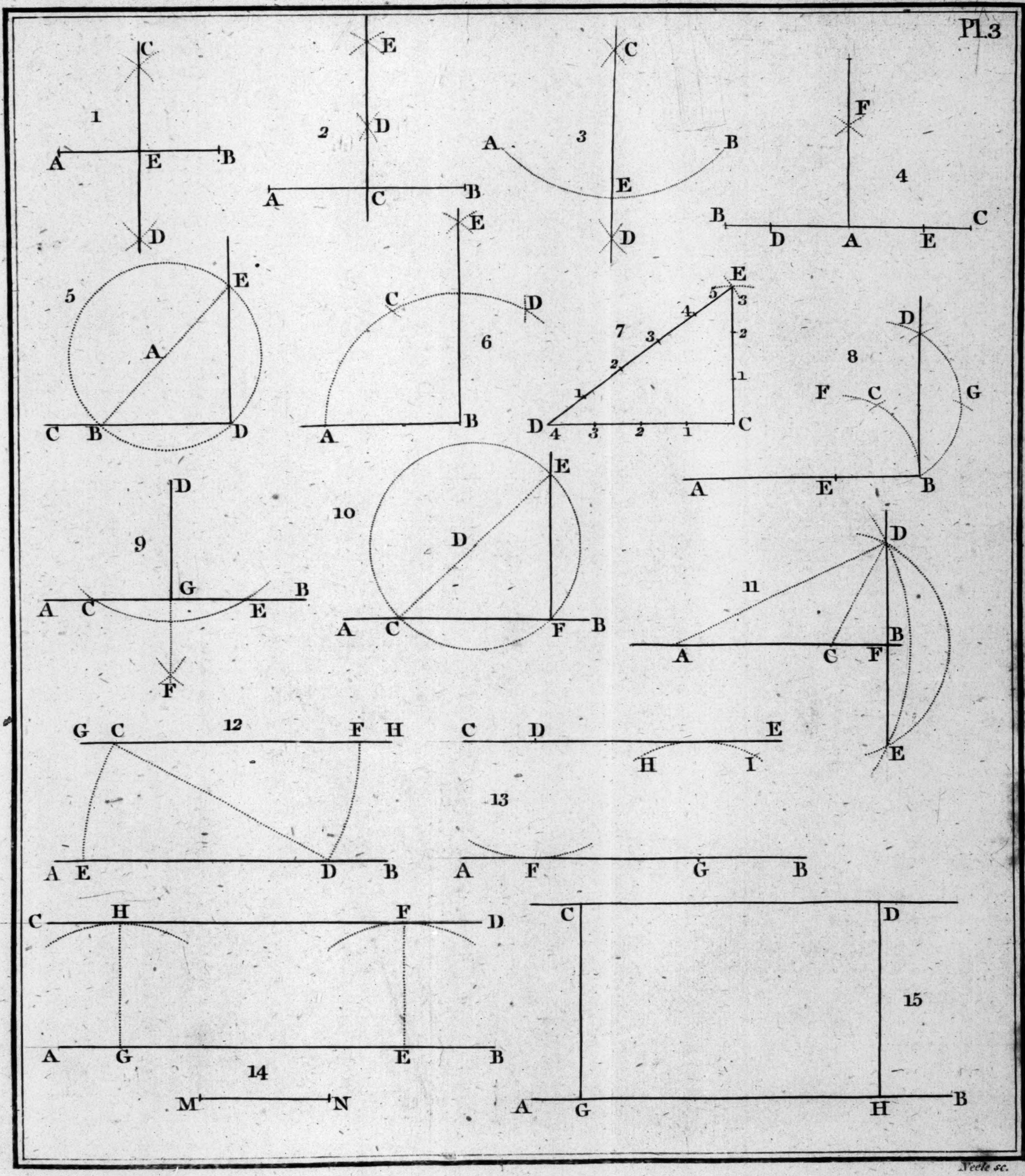
Pl. 3



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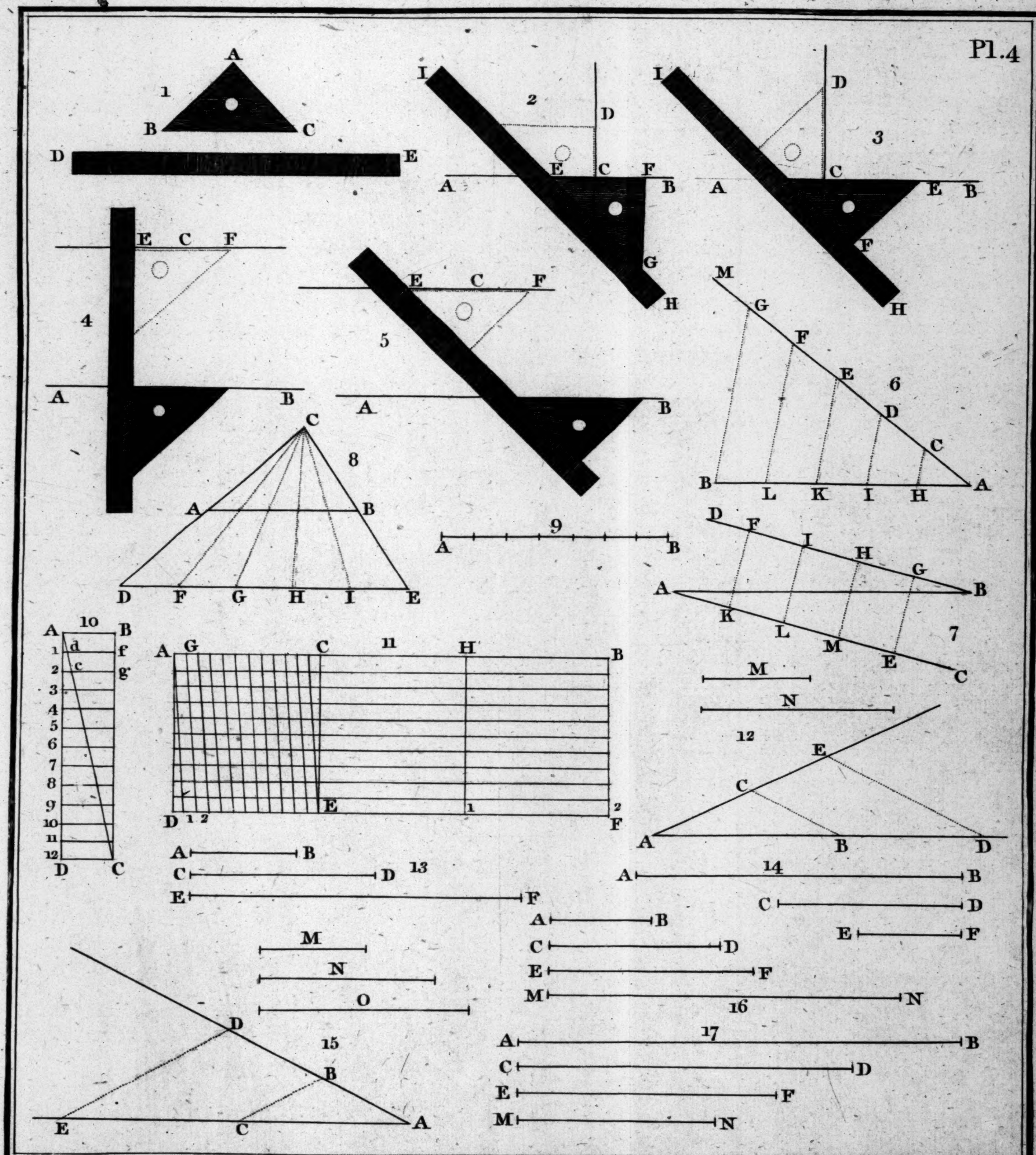

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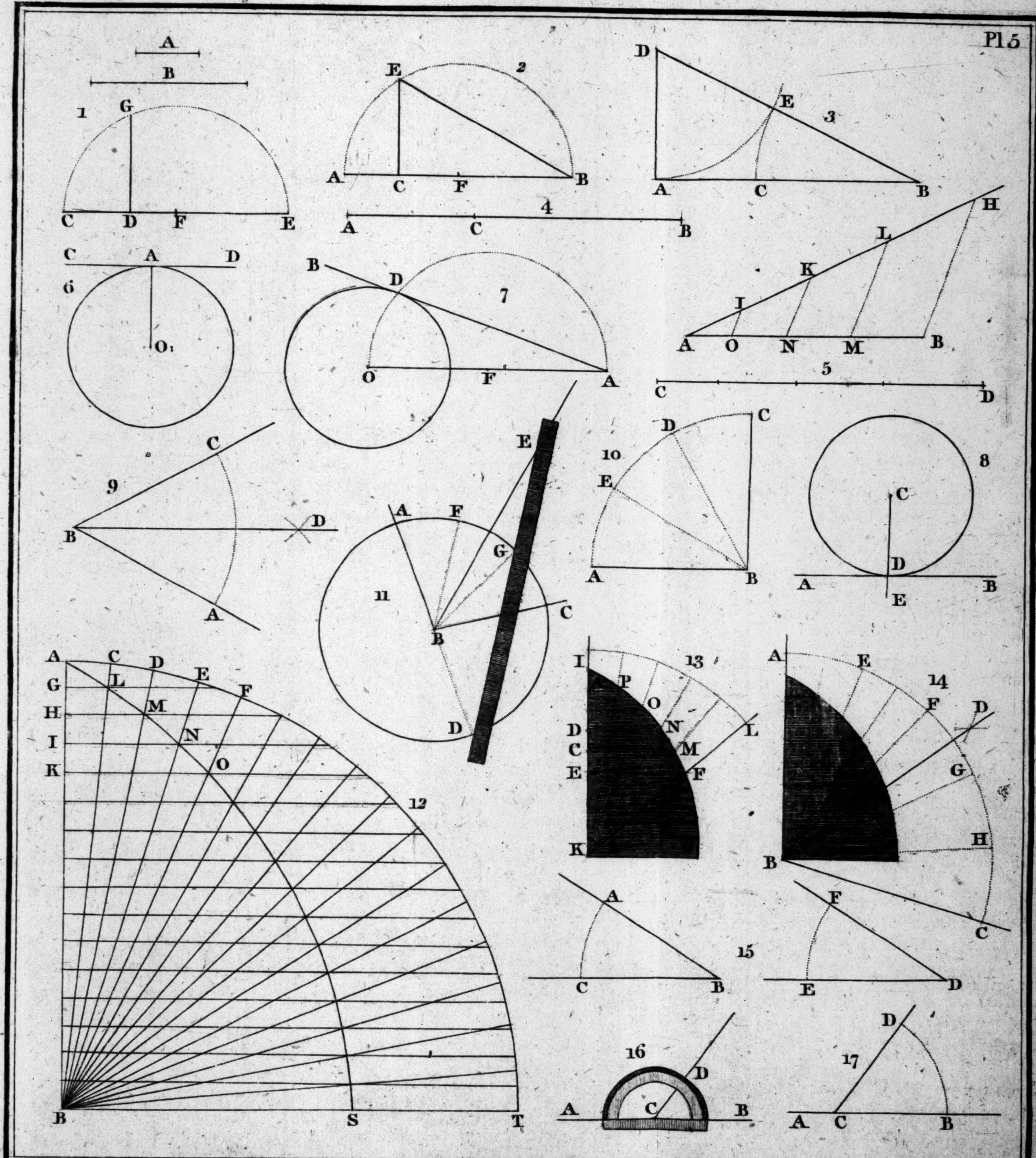

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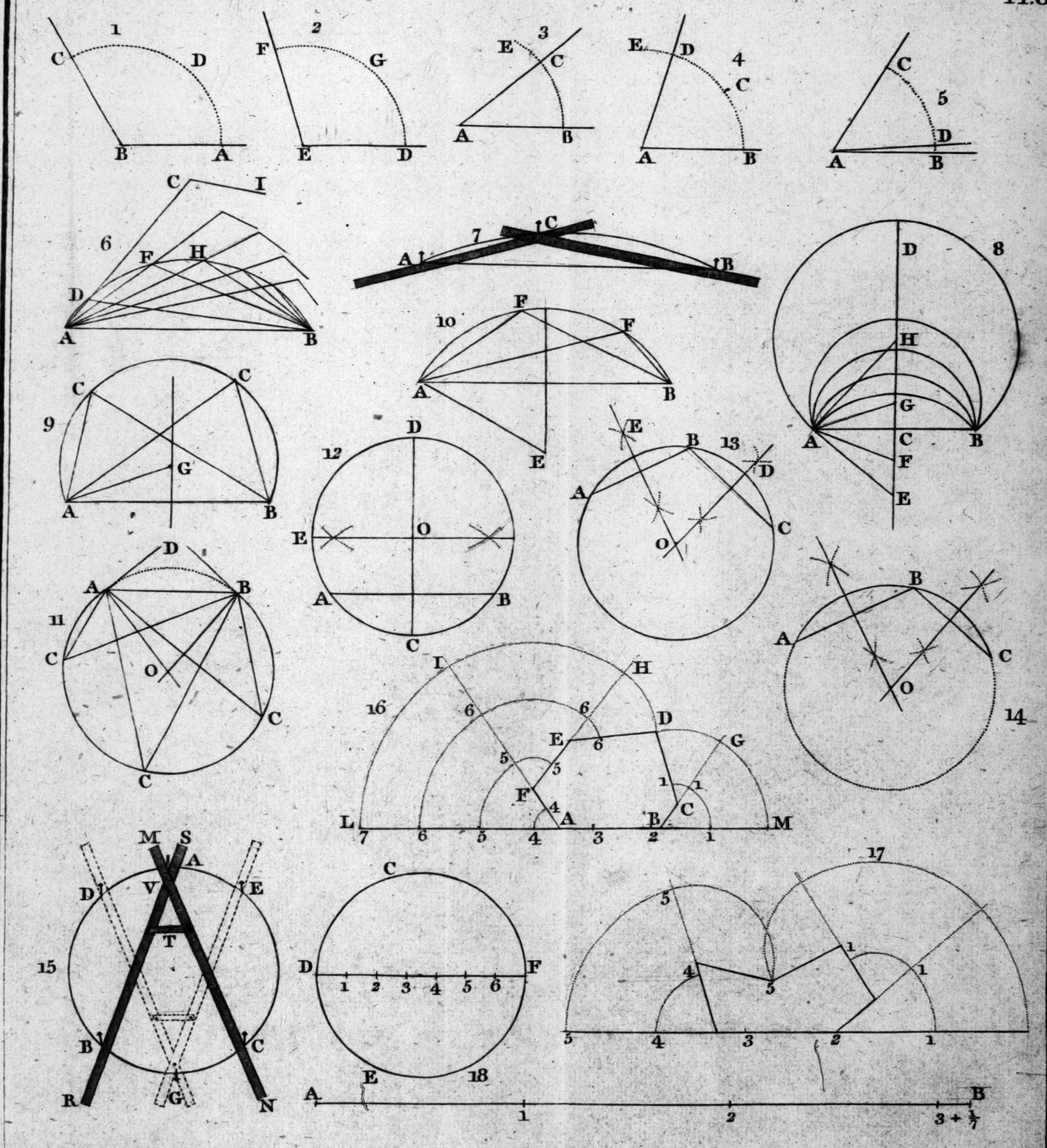
Pl. 6



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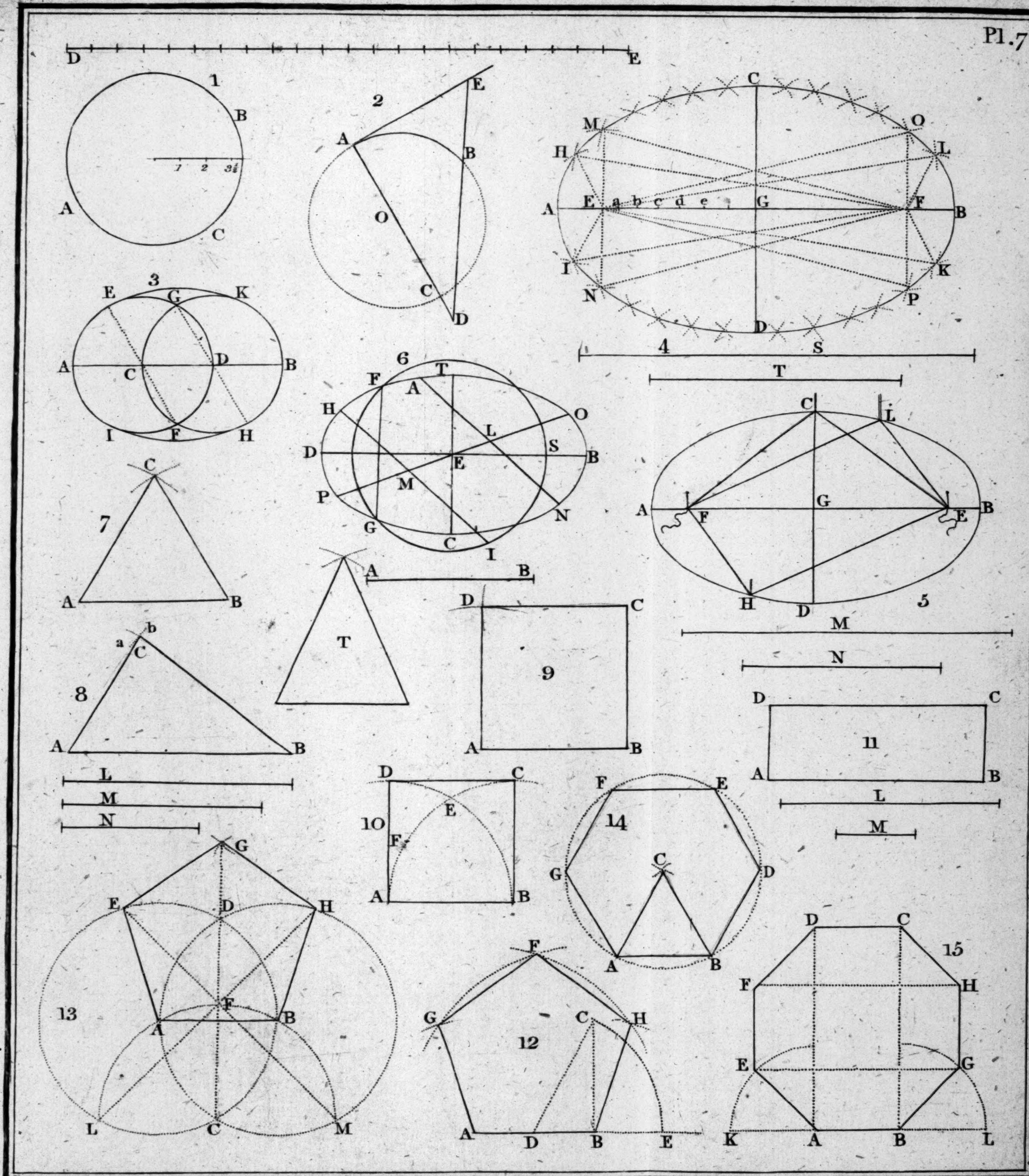
Pl. 7



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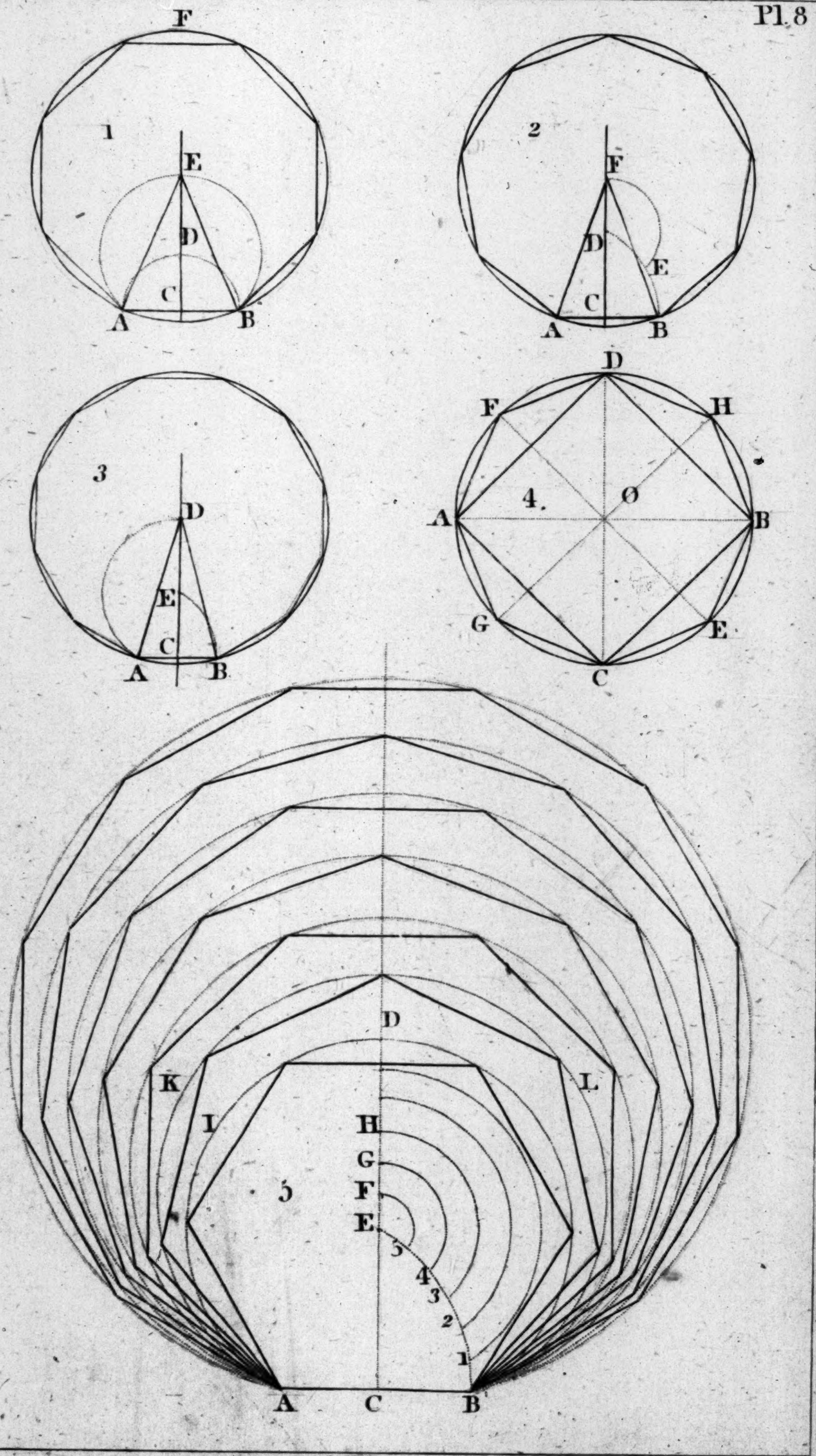
Pl. 8



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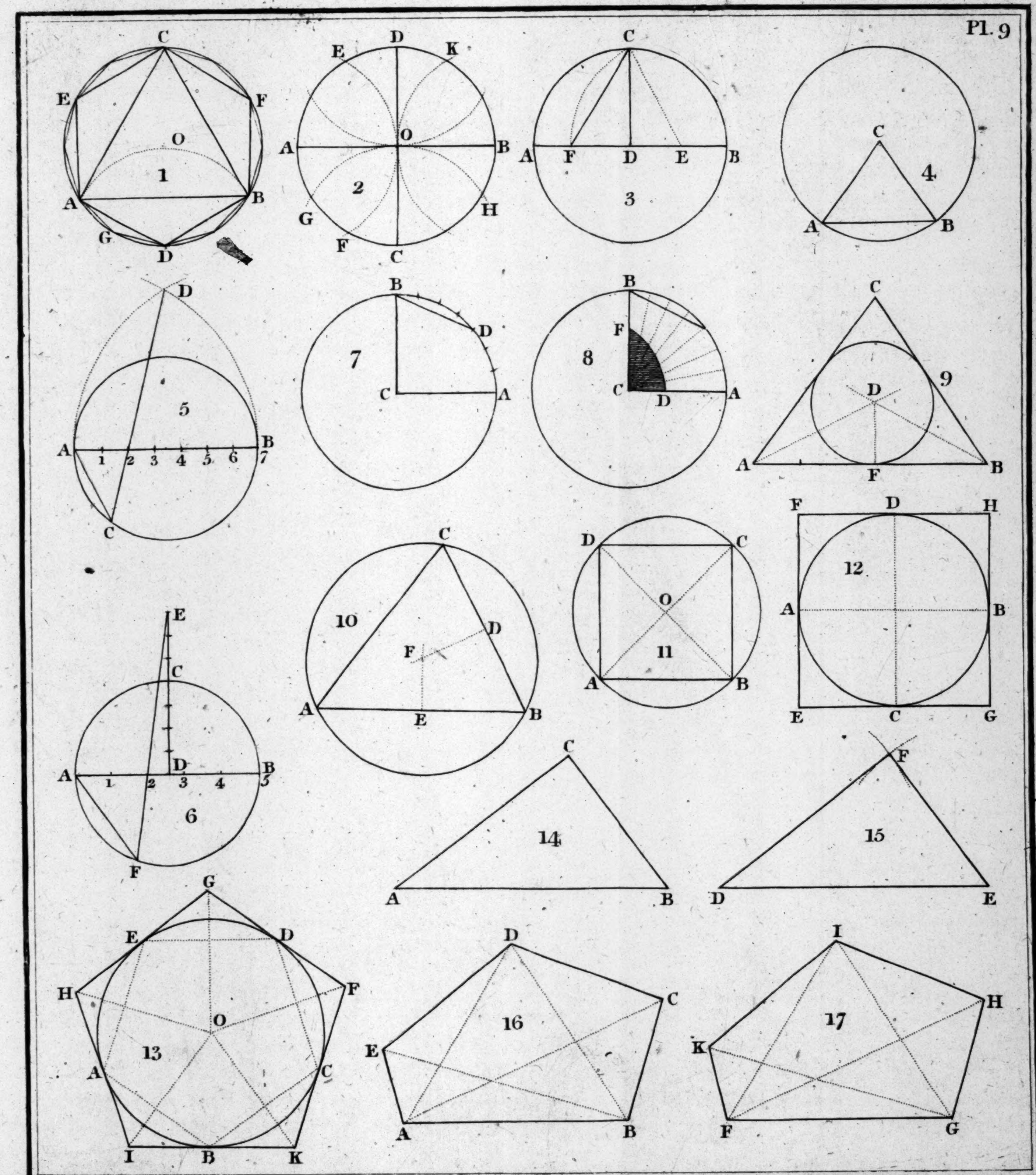

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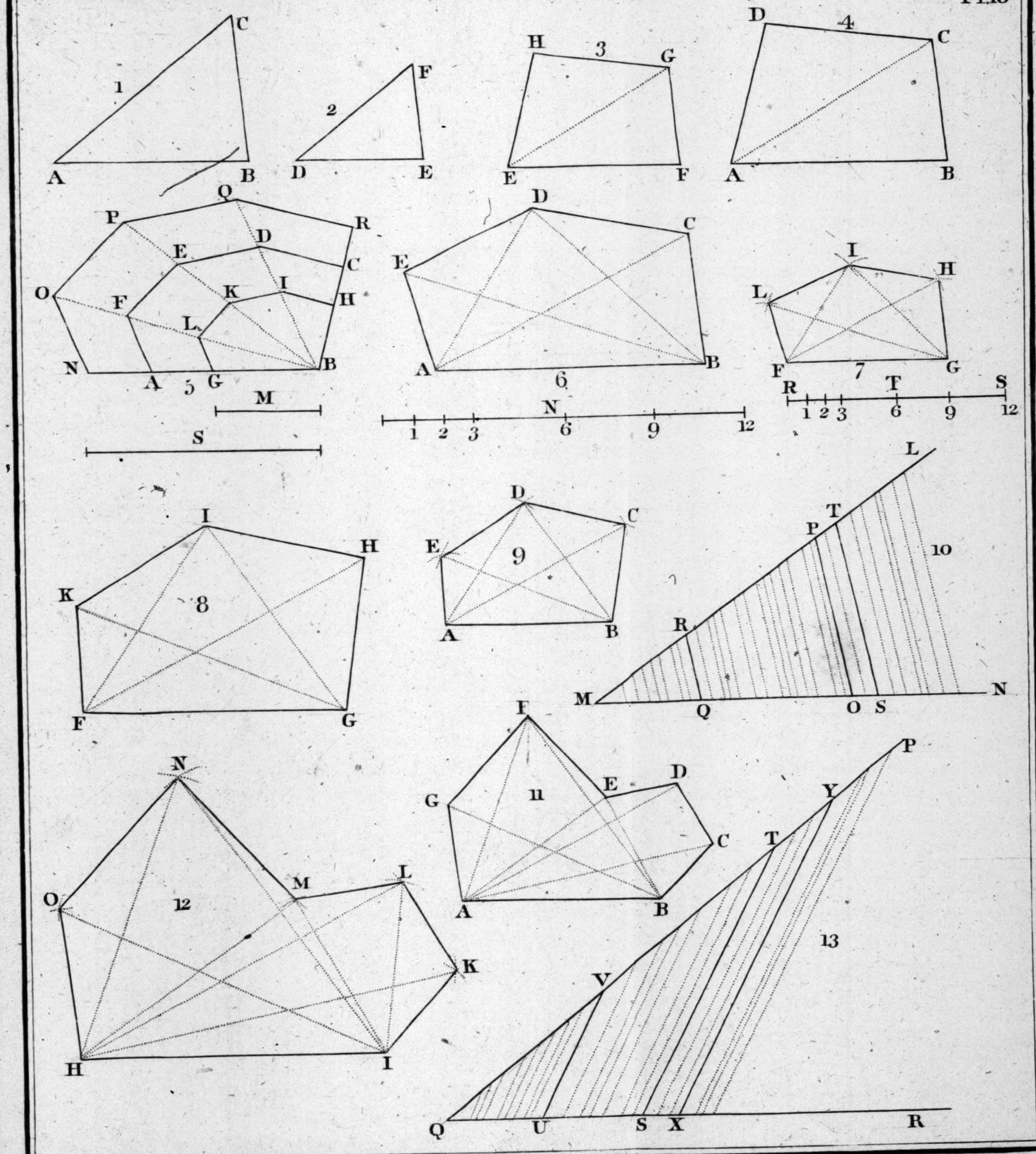
Pl.10



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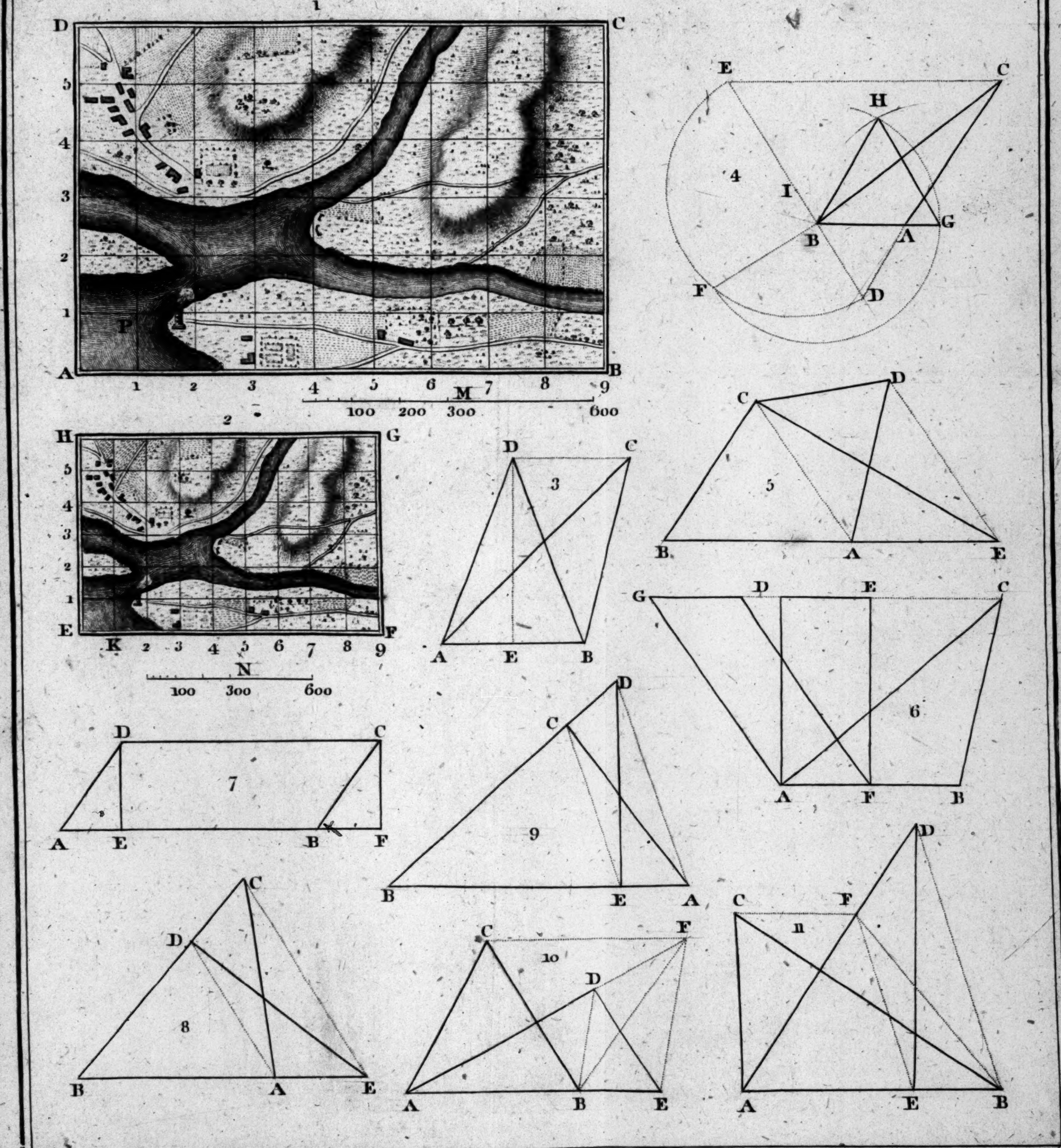
Pl. II



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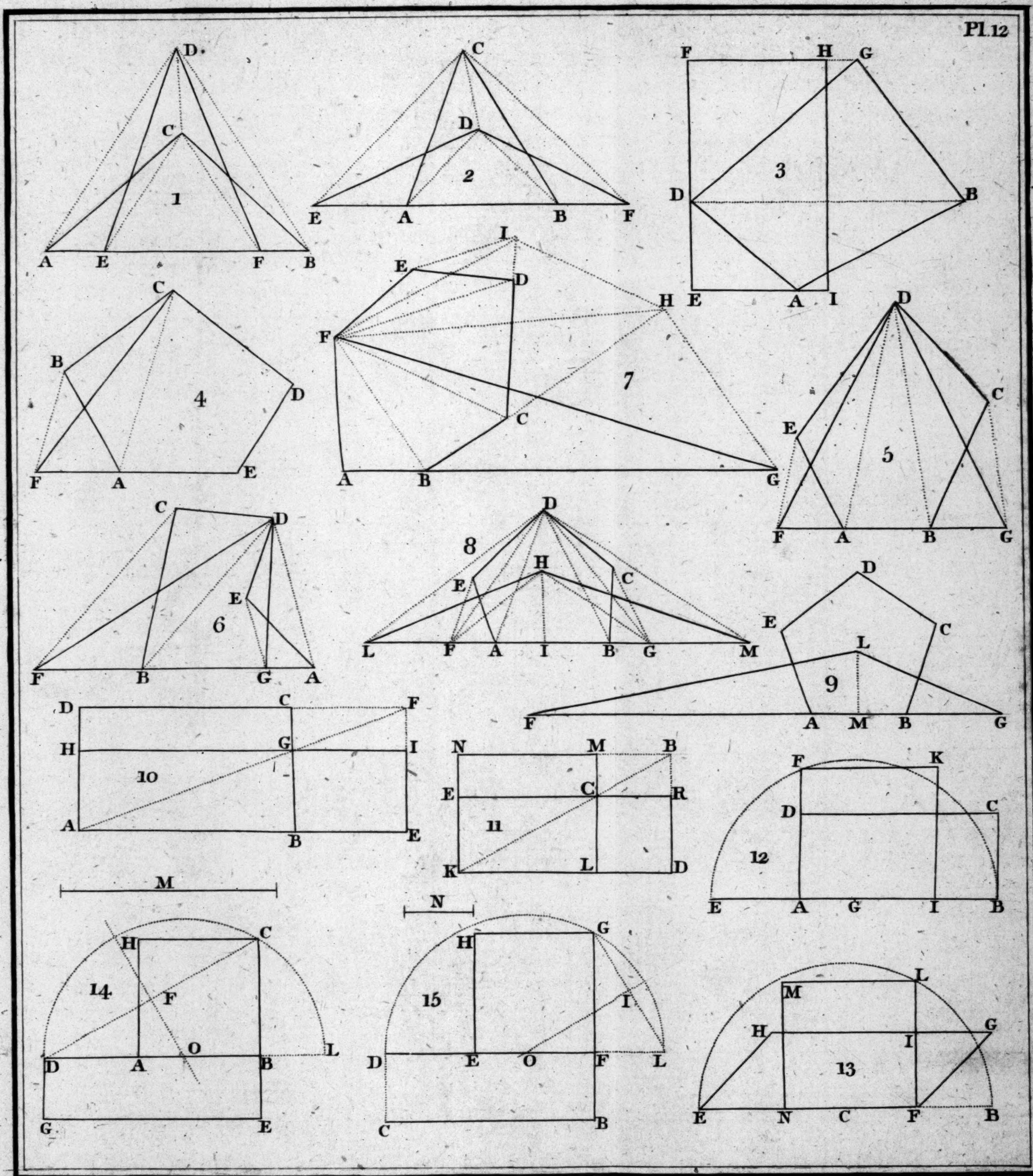

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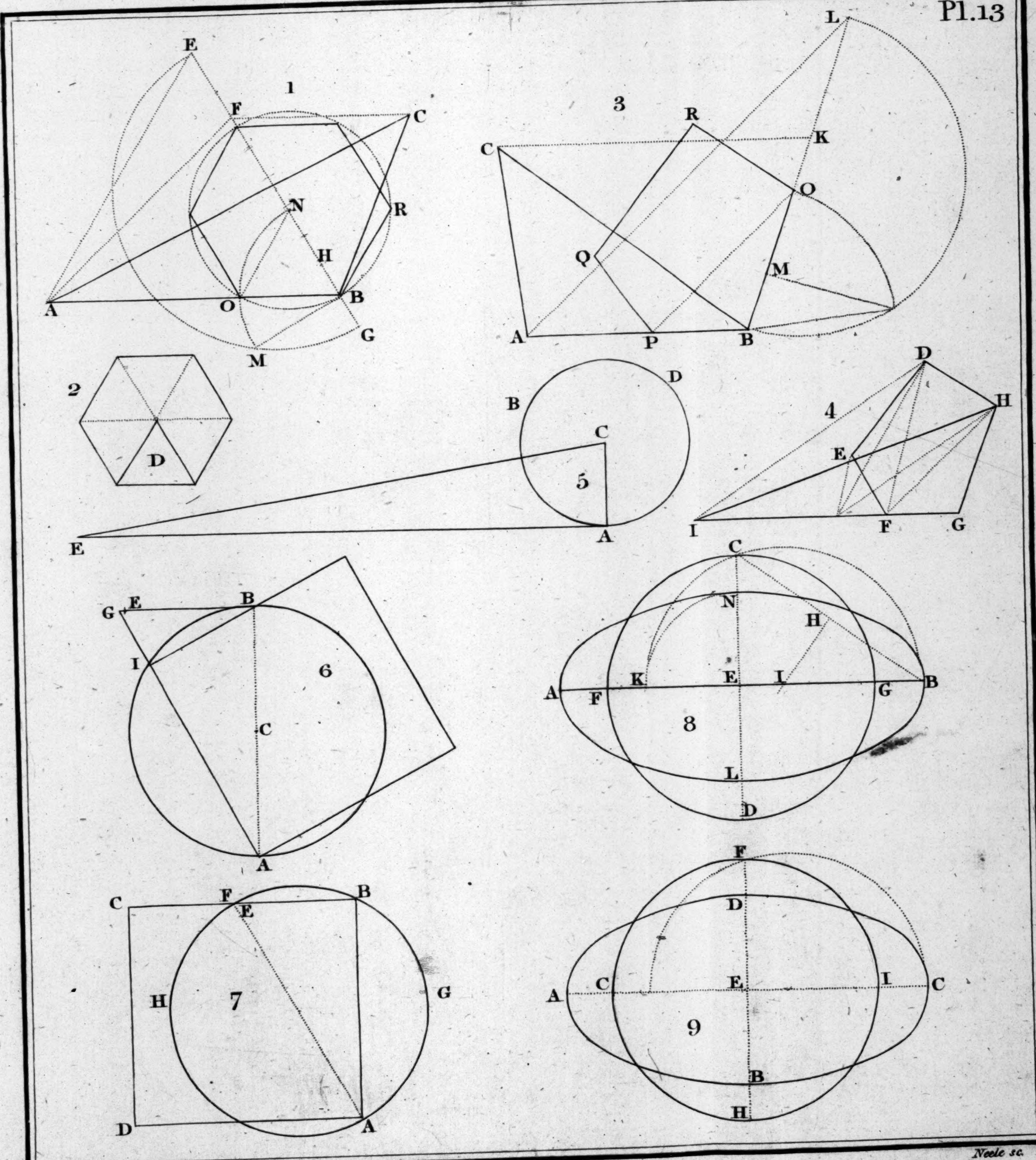
Pl. 13



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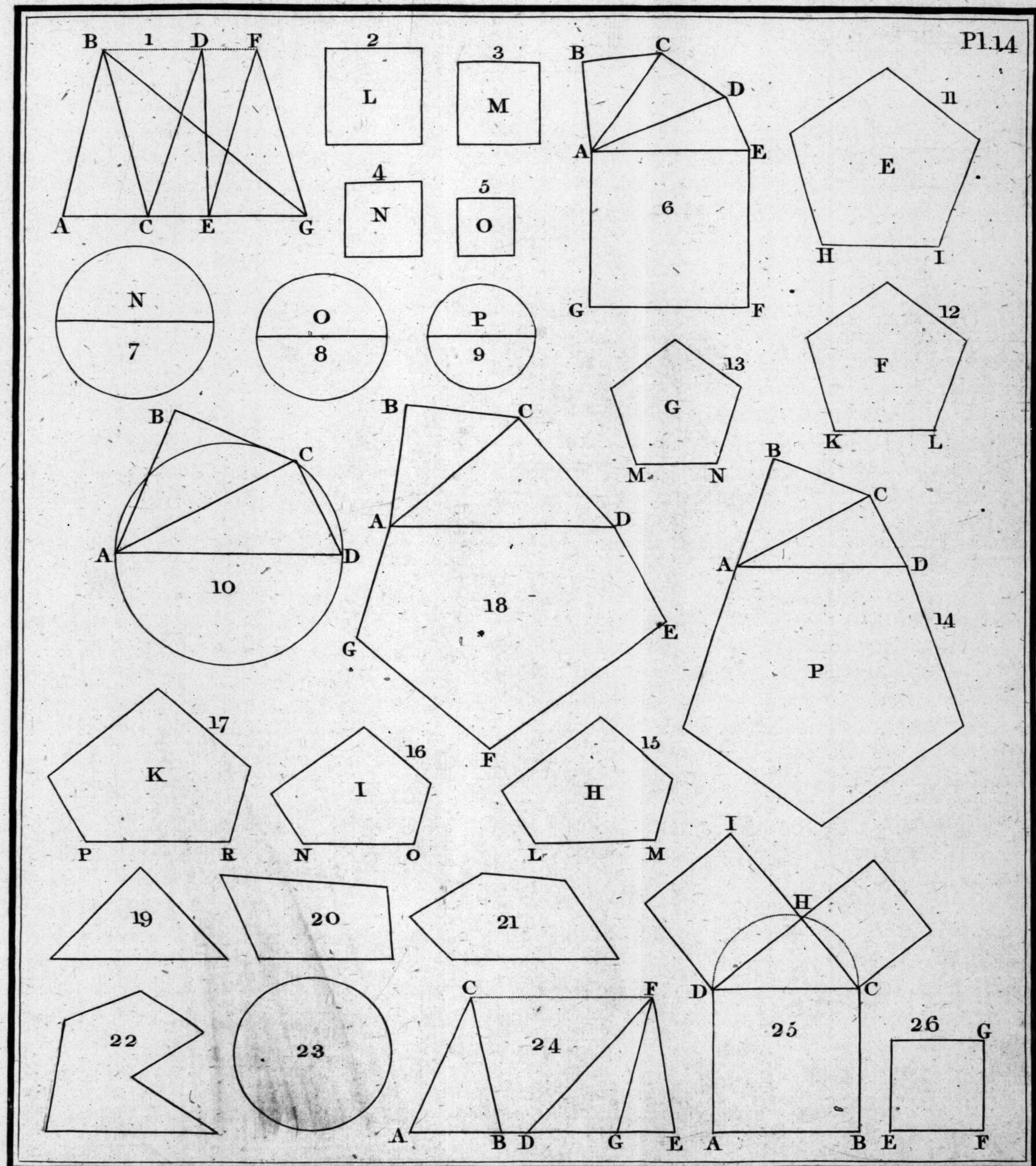

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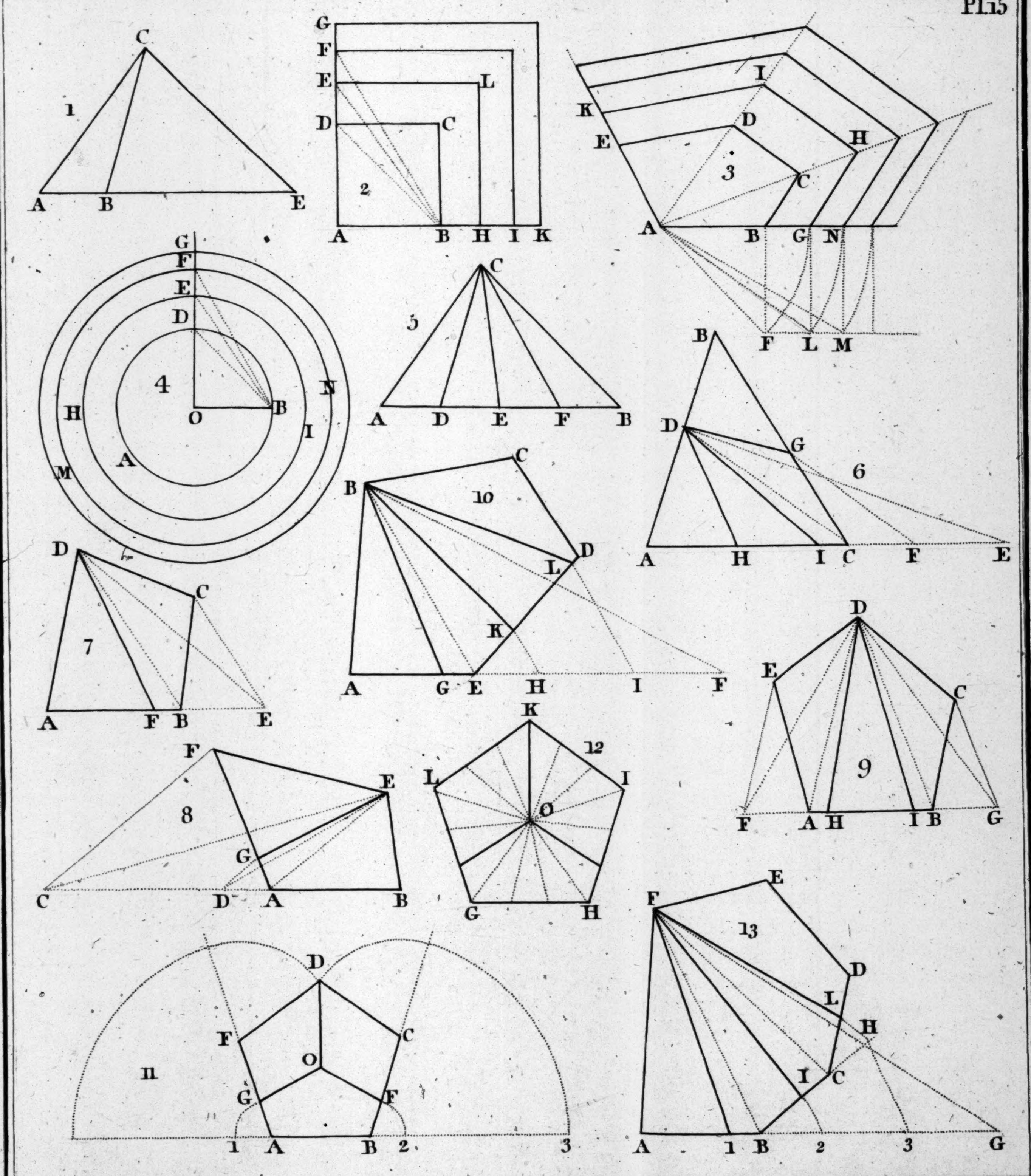
Pl.15



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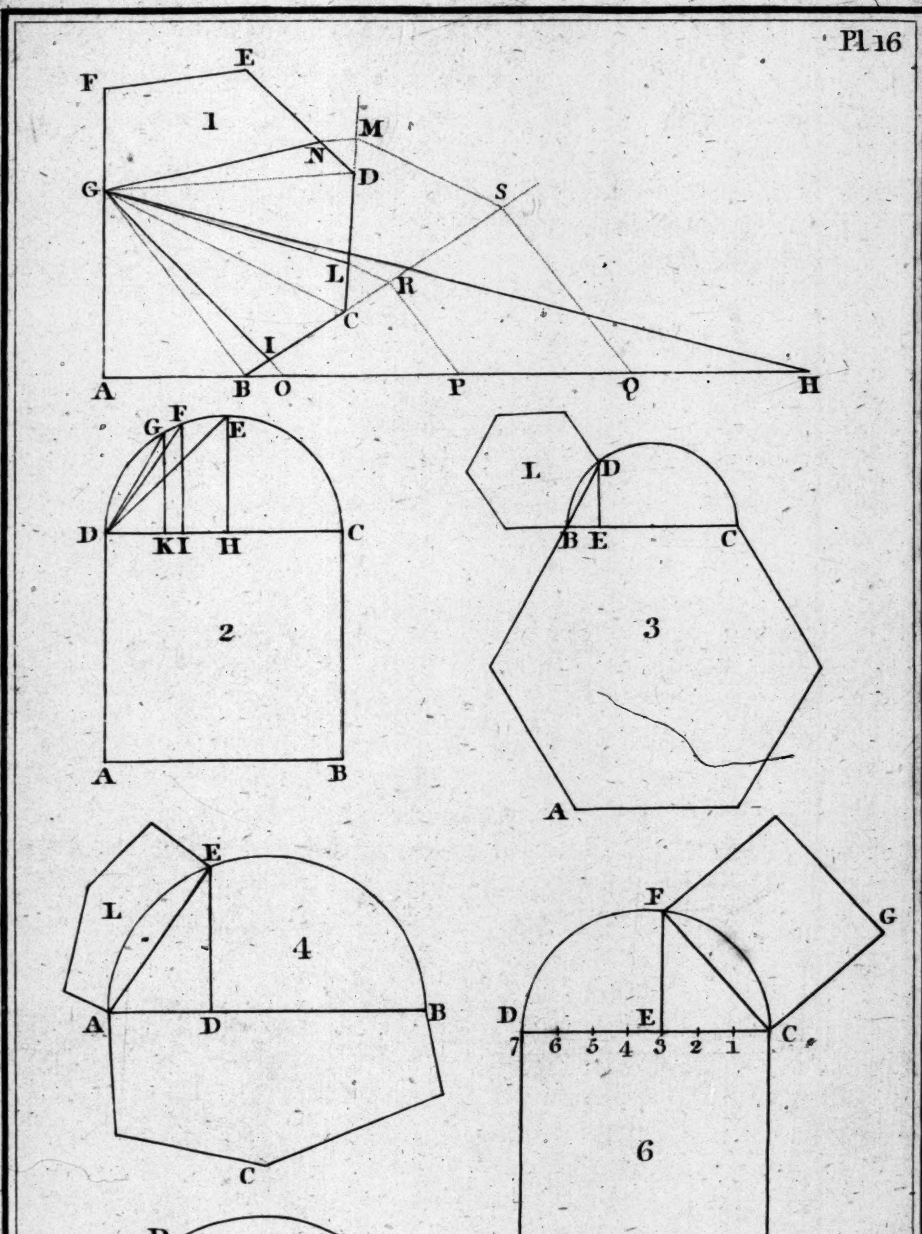
Pl. 16
1
F E M N D G S L R C I A B O P Q H
2
G F E D K I H C A B
3
L D B E C A
4
E L A D B C
6
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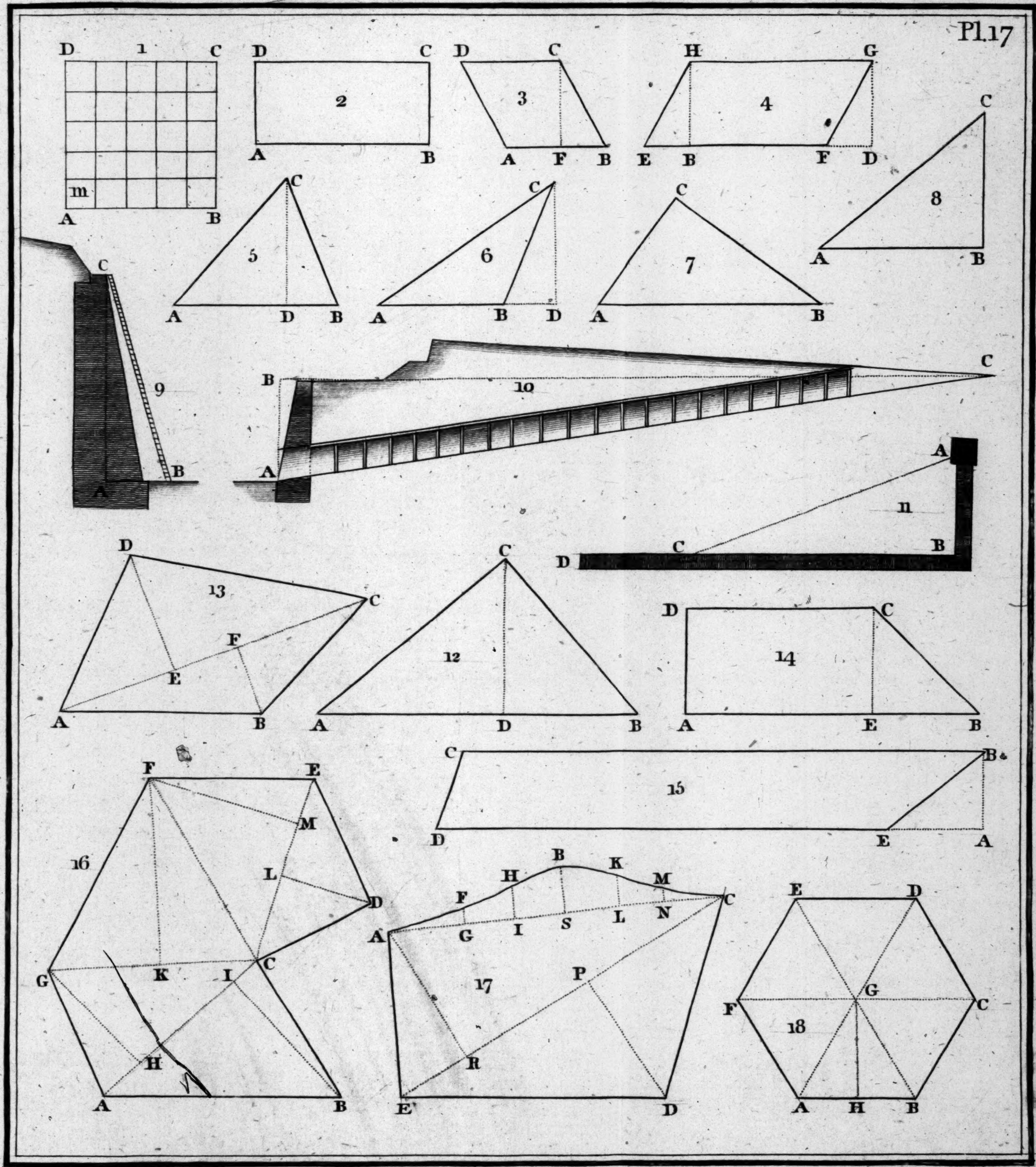
5
D F A E C
7
G H E 4 3 2 1 F

Neele sc.



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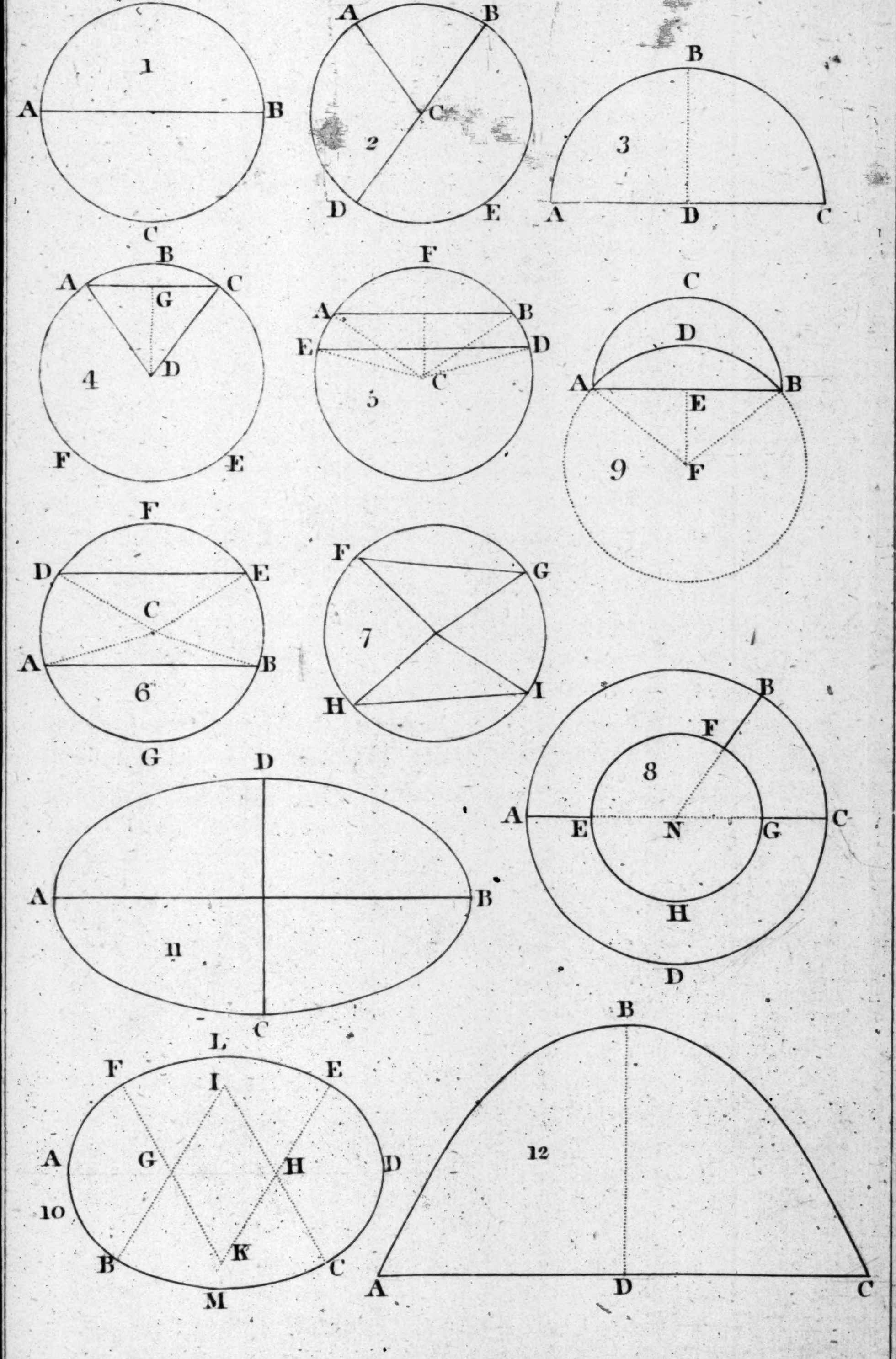
Pl. 18



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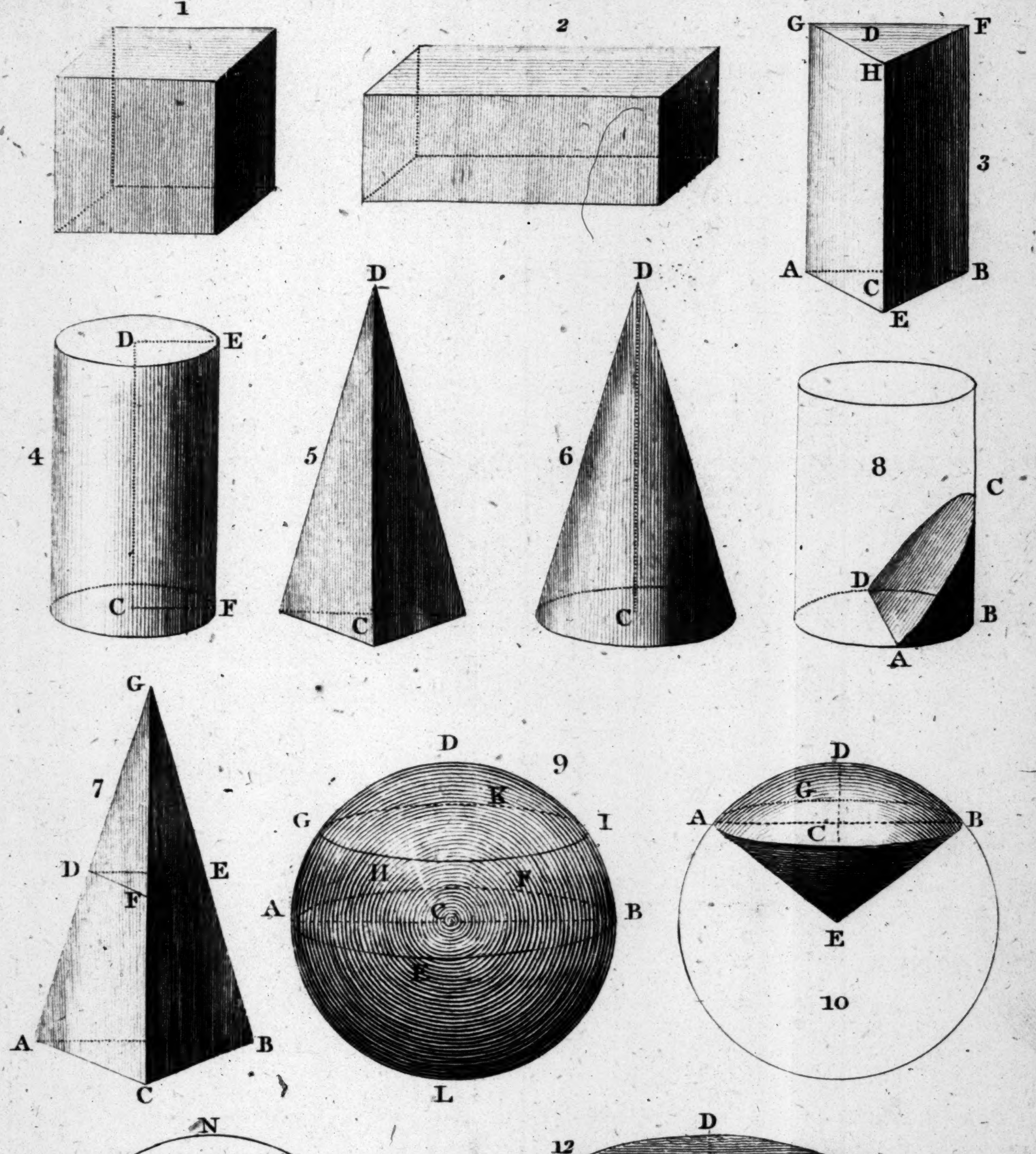

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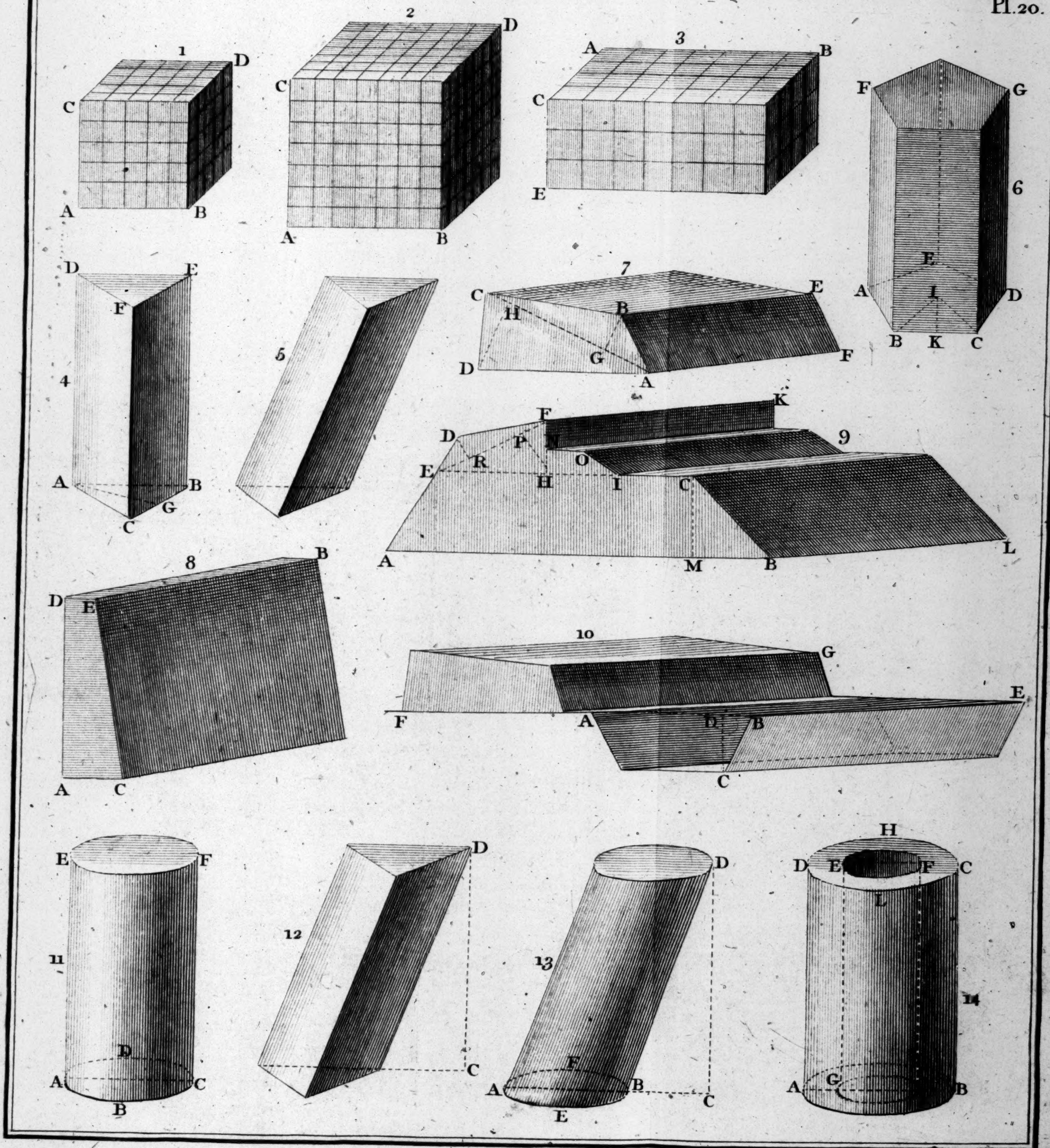
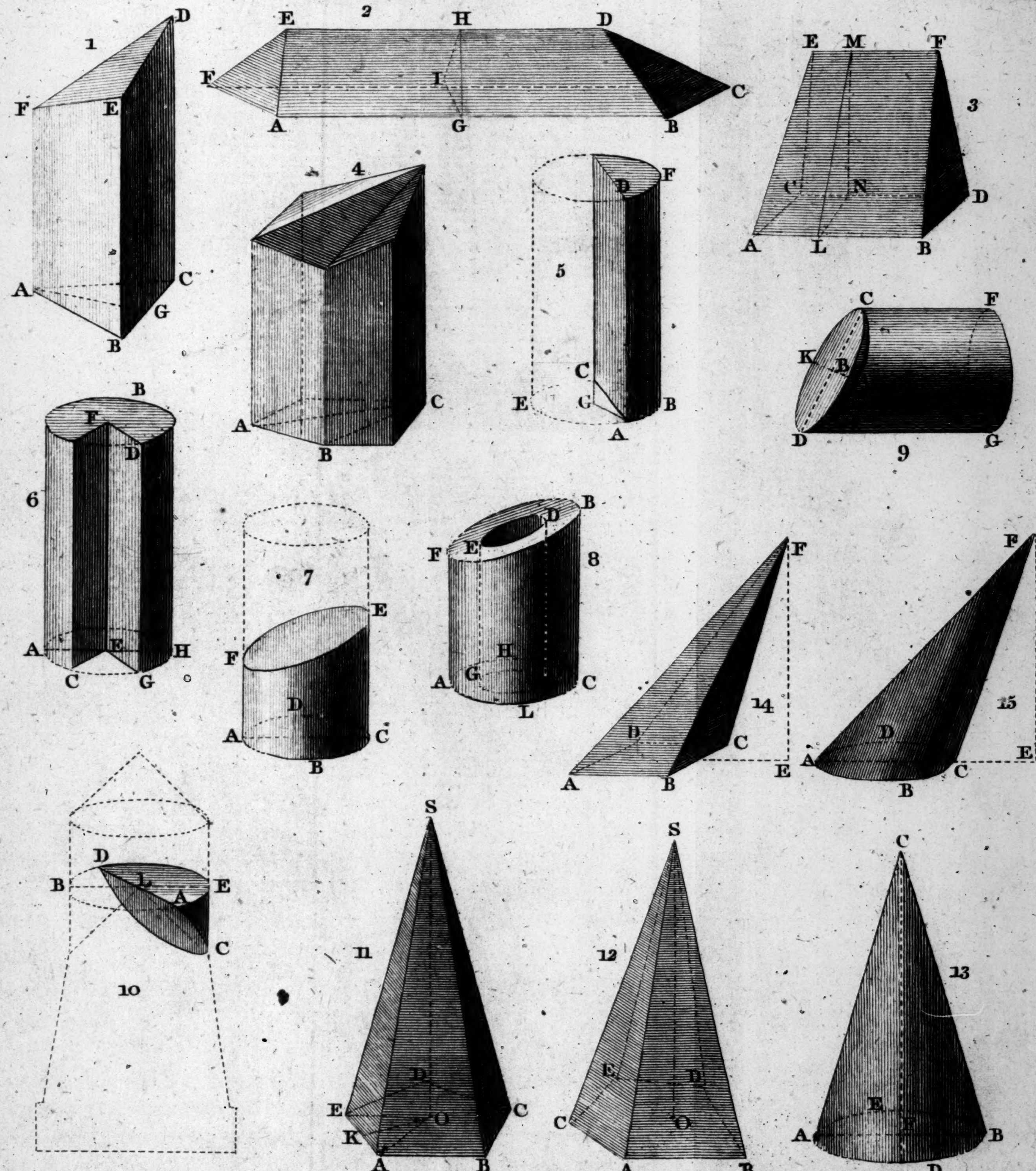
C

Pl. 20.



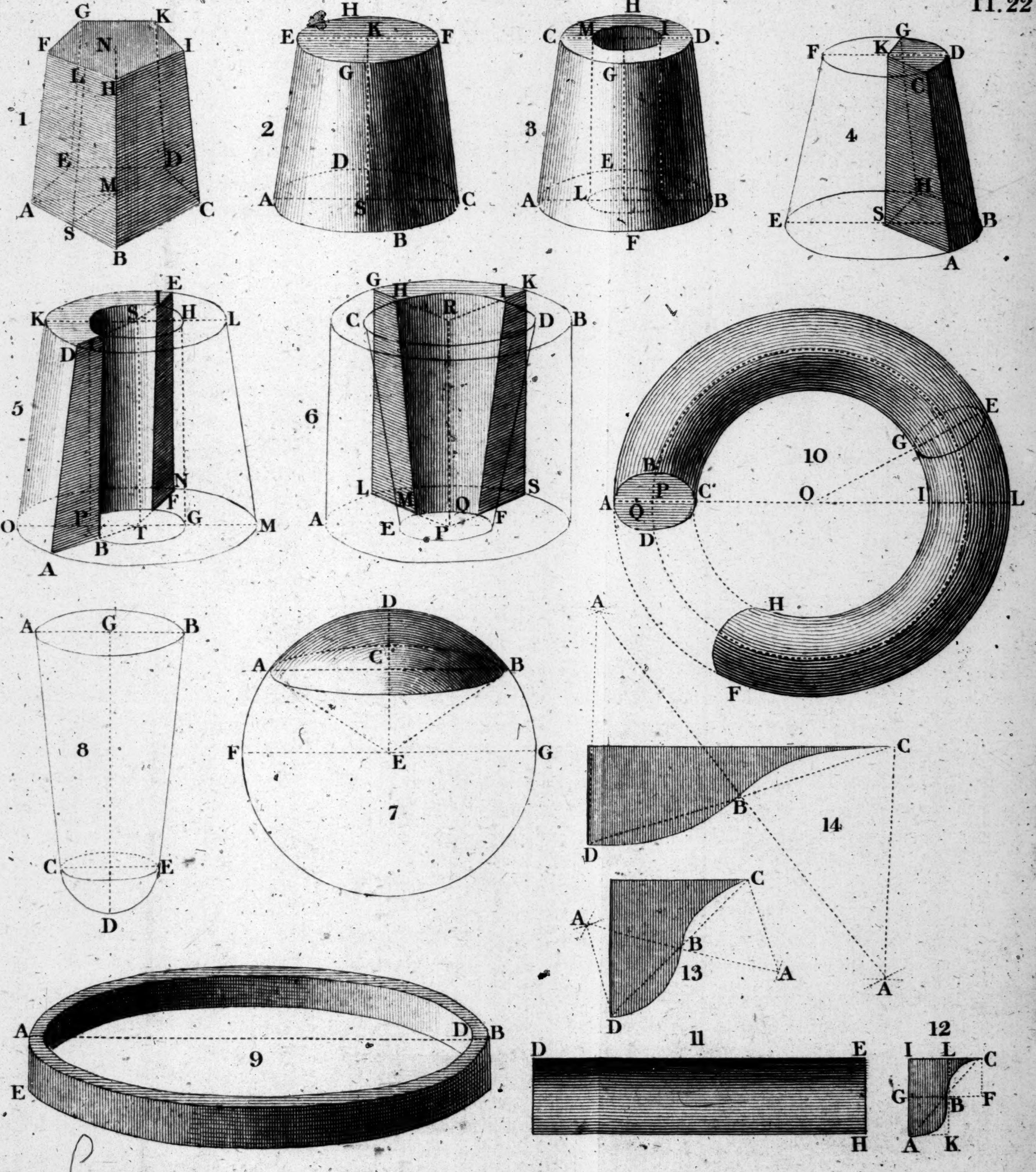
Neele sc.

Pl. 21

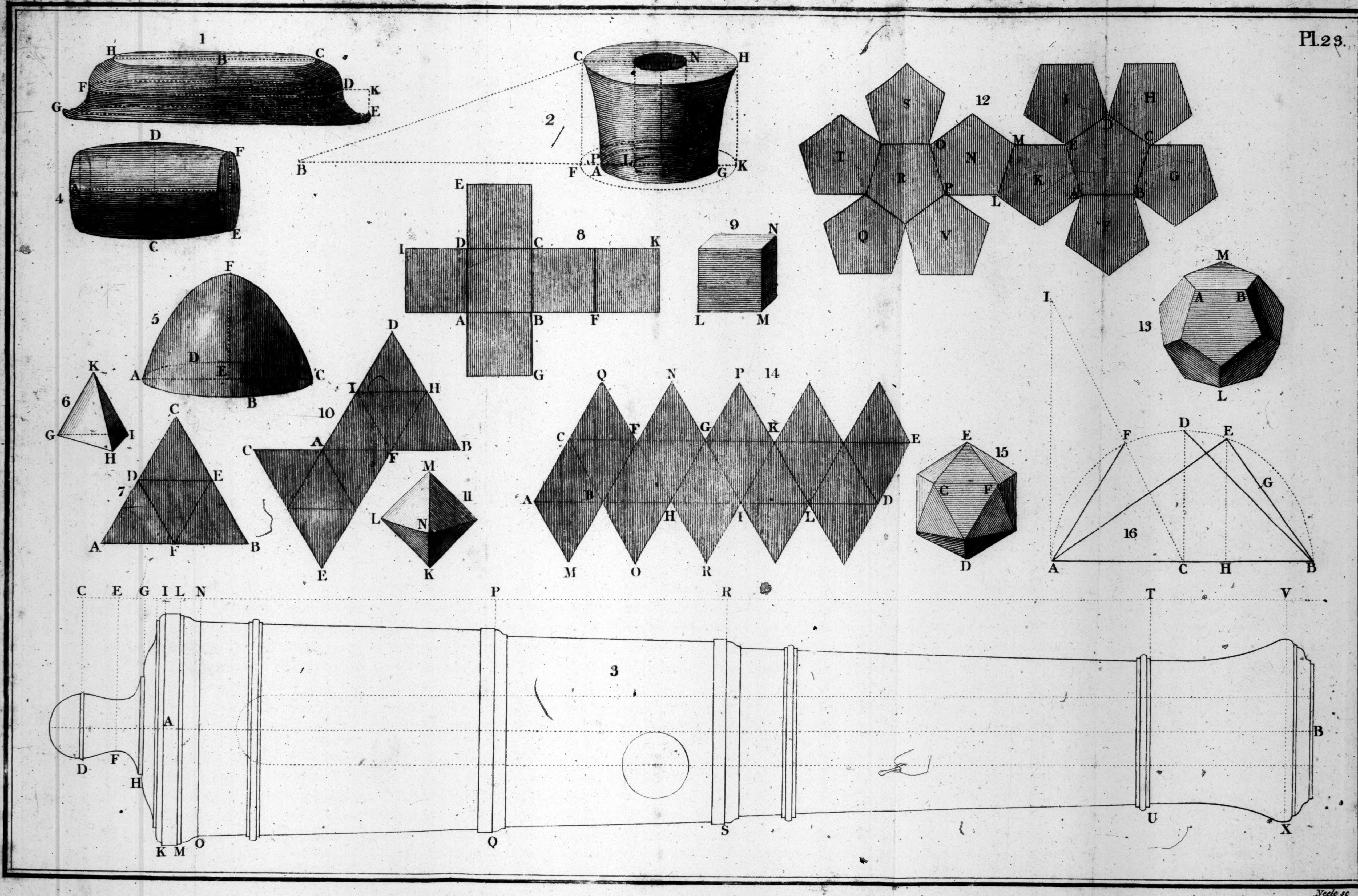


Neele sc.

Pl. 22



Neele sc.


Pl.23.
Neele sc